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著者	NOMACHI Sumio
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On one Method for Solving Three-Dimensional Stress Problems by Means of Finite Fourier Transforms

Sumio Nomachi*

Abstract

The purpose of this paper is to present the general solutions of the three-dimensional stress problems concerning a rectangular parallelepiped acted by any forces on its boundaries. The six components of the stress and the three components of the displacement are by virtue of the finite Fourier transforms written in the form of the tripple trigonometrical series with respect to x, y, and z.

Introduction

By means of the three functions approach, J. C. Maxwell[1] had first asserted and H. Neuber[2] had lately extended, or by means of Galerkin's vector which Galerkin[3] derived from the idea of Love's strain function[4], the general solutions were found out of the three-dimensional stress problems. It is, however, too difficult to determine these solutions to satisfy the boundary conditions. By that reason, the three-dimensional stress problem so far treated may mostly be connected with elastic medium wich is infinite, semiinfinite, or bodies with axially symmetrical stress. Only a few investigators[5], the author thinks, have been dealing with the three-dimensional stress problems in regard to the finite bodies which have six side planes.

The procedure herein proposed is a kind of operator calculus in which Green's formulas, relating to the equations of equilibrium of forces, are

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1) Edinburch Roy. Soc. Trans. Vol. 26 (1870). Reprinted in Maxwell's Scientific Papers.

2) H. Neuber: Kerbspannungslehre, Julius Springer, S. 19 (1937).

3) B. Galerkin: Contribution á la solution générale du probléme de la theórie de l'elasticité dans le cas de trois dimensions, Compte Rendus, 190 (1930).

4) A. E. H. Love: The Mathematical theory of elasticity, P. 262, ed. 4.

5) K. Hata: Note on one Method for Solving Three-Dimensional Stress problem, Hokkaido Univ. Kogakubu Kenkyuhokoku 11 (1954). According to Prof. T. Sato's book in which he translates "ТЕОРИЯ УПРУГОСТИ И ПЛАСТИЧНОСТИ" of БЕЗУХОВ into Japanese, the similar problem was treated in 1951 by ФИЛОНЕНКО-БОРОДИЧ М. М..

used in finding out the components of the stress and the components of the displacement in the finite Fourier transforms.

Equations of Equilibrium of Forces and Green's Formulas

The stresses acting on six sides of a cubic element of an elastic medium are expresses by three normal stresses σ_x, σ_y, and σ_z; and three shearing stresses τ_{yz}, τ_{zx}, and τ_{xy}. From the equilibrium condition applied to a cubical element of the body, the equations of equilibrium of forces take the well-known foorms

$$\frac{\partial \sigma_x}{\partial x}+\frac{\partial \tau_{xy}}{\partial y}+\frac{\partial \tau_{zx}}{\partial z}=X, \tag{1}$$

$$\frac{\partial \tau_{xy}}{\partial x}+\frac{\partial \sigma_y}{\partial y}+\frac{\partial \tau_{yz}}{\partial z}=Y, \tag{2}$$

$$\frac{\partial \tau_{zx}}{\partial x}+\frac{\partial \tau_{yz}}{\partial y}+\frac{\partial \sigma_z}{\partial z}=Z, \tag{3}$$

where X, Y, Z denote the components of the body force per unit volume in x, y, z directions respectively.

The stresses are related to the displacements u, v, and w

$$\left.\begin{aligned}
\sigma_x &= (2\mu+\lambda)\frac{\partial \mu}{\partial x}+\lambda\frac{\partial v}{\partial y}+\lambda\frac{\partial w}{\partial z},\\
\sigma_y &= \lambda\frac{\partial u}{\partial x}+(2\mu+\lambda)\frac{\partial v}{\partial y}+\lambda\frac{\partial w}{\partial z},\\
\sigma_z &= \lambda\frac{\partial u}{\partial x}+\lambda\frac{\partial v}{\partial y}+(2\mu+\lambda)\frac{\partial w}{\partial z},
\end{aligned}\right\} \tag{4}$$

$$\left.\begin{aligned}
\tau_{xy} &= \mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right),\\
\tau_{yz} &= \mu\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right),\\
\tau_{zx} &= \mu\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right),
\end{aligned}\right\} \tag{5}$$

where u, v, w, are the components of displacement in x, y, z directions respectively; and μ, λ are Lame's constants.

If we multiply the left side of Eq. (1) by L_1 that is a function differentiable two times with respect to x, y, and z; and integrate by parts, then with the following abbreviations

$$\left.\begin{aligned}
&\int_0^a\int_0^b\int_0^c f(x,y,z)\,dx\,dy\,dz=\int f(x,y,z)\,dV,\\
&\int_0^b\int_0^c f(x,y,z)\,dy\,dz=\int f(x,y,z)\,dA_x\,,\\
&\int_0^c\int_0^a f(x,y,z)\,dz\,dx=\int f(x,y,z)\,dA_y\,,\\
&\int_0^a\int_0^b f(x,y,z)\,dx\,dy=\int f(x,y,z)\,dA_z\,,
\end{aligned}\right\}\tag{6}$$

we find that

$$\begin{aligned}
&\int[\sigma_x L_1]_0^a\,dA_x+\int[\tau_{xy}L_1]_0^b\,dA_y+\int[\tau_{zx}L_1]_0^c\,dA_z\\
&\quad-\int\left\{\frac{\partial L_1}{\partial x}\,\sigma_x+\frac{\partial L_1}{\partial y}\,\tau_{xy}+\frac{\partial L_1}{\partial z}\,\tau_{zx}\right\}dV=\int XL_1\,dV
\end{aligned}$$

Putting hereinto the relations (4) and again integrating by parts, we finally have

$$\left.\begin{aligned}
&\int[\sigma_x L_1]_0^a\,dA_x+\int[\tau_{xy}L_1]_0^b\,dA_y+\int[\tau_{zx}L_1]_0^c\,dA_z-\int\Big[(2\mu+\lambda)\,u\,\frac{\partial L_1}{\partial x}\\
&\quad+\mu v\,\frac{\partial L_1}{\partial y}+\mu w\,\frac{\partial L_1}{\partial z}\Big]_0^a dA_x-\int\Big[\mu u\,\frac{\partial L_1}{\partial y}+\lambda v\,\frac{\partial L_1}{\partial x}\Big]_0^b dA_y\\
&\quad-\int\Big[\mu u\,\frac{\partial L_1}{\partial z}+\lambda w\,\frac{\partial L_1}{\partial x}\Big]_0^c dA_z+\int\Big\{\mu v\cdot\Delta L_1+(\mu+\lambda)\,u\,\frac{\partial^2 L_1}{\partial x^2}\Big\}dV\\
&\quad+(\mu+\lambda)\int\Big\{v\,\frac{\partial^2 L_1}{\partial x\partial y}+w\,\frac{\partial^2 L_1}{\partial z\partial x}\Big\}dV=\int XL_1\,dV\,,
\end{aligned}\right\}\tag{7}$$

where

$$\Delta=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\,.$$

Similary, Eqs. (2) and (3), yield the following Green's formulas:

$$\left.\begin{aligned}
&\int[\tau_{xy}L_2]_0^a\,dA_x+\int[\sigma_y L_2]_0^b\,dA_y+\int[\tau_{yz}L_2]_0^c\,dA_z-\int\Big[\mu u\,\frac{\partial L_2}{\partial x}\\
&\quad+(2\mu+\lambda)\,v\,\frac{\partial L_2}{\partial y}+\mu w\,\frac{\partial L_2}{\partial z}\Big]_0^b dA_y-\int\Big[\mu v\,\frac{\partial L_2}{\partial x}+\lambda u\,\frac{\partial L_2}{\partial y}\Big]_0^a dA_x\\
&\quad-\int\Big[\mu v\,\frac{\partial L_2}{\partial z}+\lambda w\,\frac{\partial L_2}{\partial y}\Big]_0^c dA_z+\int\Big\{\mu v\cdot\Delta L_2+(\mu+\lambda)\,v\,\frac{\partial^2 L_2}{\partial y^2}\Big\}dV\\
&\quad+(\mu+\lambda)\int\Big\{w\,\frac{\partial^2 L_2}{\partial y\partial z}+u\,\frac{\partial^2 L_2}{\partial x\partial y}\Big\}dV=\int YL_2\,dV\,,
\end{aligned}\right\}\tag{8}$$

$$\left.\begin{aligned}
&\int[\tau_{zx}L_3]_0^a\,dA_x+\int[\tau_{yz}L_3]_0^b\,dA_y+\int[\sigma_z L_3]_0^c\,dA_z-\int\Big[\mu u\,\frac{\partial L_3}{\partial x}\\
&\quad+v\,\frac{\partial L_3}{\partial y}+(2\mu+\lambda)\,w\,\frac{\partial L_3}{\partial z}\Big]_0^c dA_z-\int\Big[\mu w\,\frac{\partial L_3}{\partial x}+\lambda u\,\frac{\partial L_3}{\partial z}\Big]_0^a dA_x
\end{aligned}\right\}\tag{9}$$

$$\left.\begin{aligned}&-\iint\left[\mu w \frac{\partial L_3}{\partial y}+\lambda v \frac{\partial L_3}{\partial z}\right]_0^b dA_y+\int\left\{\mu w\cdot\Delta L_3+(\mu+\lambda)\, w \frac{\partial^2 L_3}{\partial z^2}\right\} dV \\ &+(\mu+\lambda)\int\left\{u \frac{\partial^2 L_3}{\partial z \partial x}+v \frac{\partial^2 L_3}{\partial y \partial z}\right\} dV=\int Z L_3\, dV .\end{aligned}\right\}$$

where L_2 and L_3 are each a function which can be differentiated two times with respect to x, y, and z.

Nwe Simbolic Notations

If $f(x)$ satisfies Drichlet's conditions in the interval $(0, a)$ and if for that range its finite sine and cosine transform are defined to be

$$S_m[f(x)]=\int_0^a f(x) \sin \frac{m\pi}{a} x\, dx, \tag{10}$$

$$C_m[f(x)]=\int_0^a f(x) \cos \frac{m\pi}{a} x\, dx, \tag{11}$$

then, at each point of $(0, a)$ at which $f(x)$ is continious,

$$f(x)=\frac{2}{a} \sum_m S_m[f(x)]\cdot \sin \frac{m\pi}{a} x, \tag{12}$$

$$f(x)=\frac{1}{a} \int_0^a f(x)\, dx+\frac{2}{a} \sum_m C_m[f(x)]\cdot \cos \frac{m\pi}{a} x, \tag{13}$$

which can be extended to functions of three variables. Suppose, for instance, that $f(x, y, z)$ is a function of the three independent variables, x, y, and z, and satisfies Drichilet's conditions in the spacial domain $(0<x<a$, $0<y<b$, $0<z<c)$; then the finite tripple Fourier transform of $f(x, y, z)$ with regard to $\sin m\pi x/a$, $\sin n\pi y/b$, and $\sin r\pi z/c$, may be written in

$$S_m S_n S_r[f(x, y, z)]=\int_0^a \int_0^b \int_0^c f(x, y, z)\cdot \sin \frac{m\pi}{a} x\cdot \sin \frac{n\pi}{b} y\cdot \sin \frac{r\pi}{c} z\, dx\, dy\, dz, \tag{14}$$

where

$$m, n, r=1, 2, 3, 4, \cdots\cdots,$$

then, at eachp oint of that space at which $f(x, y, z)$ is continious.

$$f(x, y, z)=\frac{8}{abc} \sum_m \sum_n \sum_r S_m S_n S_r[f(x, y, z)]\cdot \sin \frac{m\pi}{a} x \sin \frac{n\pi}{b} y \sin \frac{r\pi}{c} z . \tag{15}$$

In a similar way, the finite tripple Fourier transforms with respect to $\sin m\pi x/a \cos n\pi y/b \cos r\pi z/c$ and $\cos m\pi x/a \sin n\pi y/b \cos r\pi z/c$, are related with $f(x, y, z)$ by the two inversion formulas:

$$
\left.
\begin{aligned}
f(x, y, z) &= \frac{2}{abc} \sum_m \sin \frac{m\pi}{a} x \left\{ \int S_m [f(x, y, z)]\, dA_x \right. \\
&+ 2 \sum_n \int_0^c S_m C_n [f(x, y, z)]\, dz \cdot \cos \frac{n\pi}{b} y + 2 \sum_r \int_0^b S_m C_r [f(x, y, z)]\, dy \\
&\times \cos \frac{r\pi c}{z} + 4 \sum_n \sum_r S_m C_n C_r [f(x, y, z)] \cdot \cos \frac{n\pi}{b} y \cdot \cos \frac{r\pi}{c} z \left. \right\},
\end{aligned}
\right\} \quad (16)
$$

$$
\left.
\begin{aligned}
f(x, y, z) &= \frac{2}{abc} \sum_n \sin \frac{n\pi}{b} y \left\{ \int S_n [f(x, y, z)]\, dA_y \right. \\
&+ 2 \sum_m \int_0^c C_m S_n [f(x, y, z)]\, dz \times \cos \frac{m\pi}{a} x + 2 \sum_r \int_0^a S_n C_r [f(x, y, z)]\, dx \\
&\times \cos \frac{r\pi}{c} z + 4 \sum_m \sum_r C_m S_n C_r [f(,x, y, z)] \cdot \cos \frac{m\pi}{a} x \cdot \cos \frac{r\pi}{c} z \left. \right\}.
\end{aligned}
\right\} \quad (17)
$$

Solutions of Rectangular Parallelopiped

If we locate the rectangular Cartesian coordinate as snown in Fig. 1.

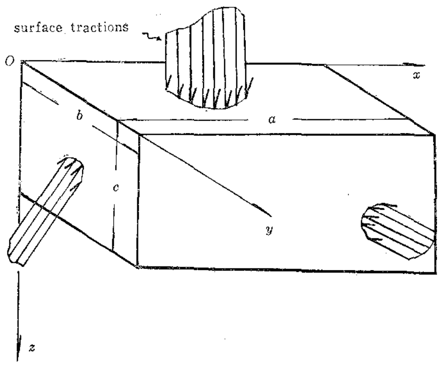

Fig. 1.

and let

$$
\left.
\begin{aligned}
L_1 &= \cos \frac{m\pi}{a} x \cdot \sin \frac{n\pi}{b} y \cdot \cos \frac{r\pi}{c} z, \\
L_2 &= \sin \frac{m\pi}{a} x \cdot \cos \frac{n\pi}{b} y \cdot \cos \frac{r\pi}{c} z, \\
L_3 &= \sin \frac{m\pi}{a} x \cdot \sin \frac{n\pi}{b} y \cdot \sin \frac{r\pi}{c} z,
\end{aligned}
\right\} \quad (18)
$$

then, with the notations

$$
M = \frac{m\pi}{a}, \quad N = \frac{n\pi}{b}, \quad R = \frac{r\pi}{c},
$$

Eq. (7) becomes

for $n \neq m = r = 0$,

$$\left.\begin{aligned}&\int_0^c S_n\,[(\sigma_x)_{x=a}]\,dz - \int_0^c S_n\,[(\sigma_x)_{x=0}]\,dz + \int_0^a S_n\,[(\tau_{zx})_{z=c}]\,dx \\ &\quad - \int_0^a S_n\,[(\tau_{zx})_{z=0}]\,dx - \mu N\left\{\int_0^c C_n\,[v_{x=a}]\,dz - \int_0^c C_n\,[v_{x=0}]\,dz\right\} \\ &\quad - \mu N\left\{(-1)^n\int(u_{y=b})\,dA_y - \int(u_{y=0})\,dA_y\right\} - \mu N^2\int S_n\,[u]\,dA_y = \int S_n\,[X]\,dA_y\end{aligned}\right\} \tag{19}$$

for $m,\ n \neq 0,\ r = 0$,

$$\left.\begin{aligned}&(-1)^m\int_0^c S_n\,[(\sigma_x)_{x=a}]\,dz - \int_0^c S_n\,[(\sigma_x)_{x=0}]\,dz + C_m S_n\,[(\tau_{zx})_{z=c}] \\ &\quad - C_m S_n\,[(\tau_{zx})_{z=0}] - \mu N\left\{(-1)^m\int_0^c C_n\,[v_{x=a}]\,dz - \int_0^c C_n\,[v_{x=0}]\,dz\right\} \\ &\quad - \mu N\left\{(-1)^n\int_0^c C_m\,[u_{y=b}]\,dz - \int_0^c C_m\,[u_{y=0}]\,dz\right\} - \left\{M^2(2\mu+\lambda) + N^2\mu\right\} \\ &\quad \times \int_0^c C_m S_n\,[u]\,dz - MN(\mu+\lambda)\int_0^c S_m C_n\,[v]\,dz = \int_0^c C_m S_n\,[X]\,dz\end{aligned}\right\} \tag{20}$$

for $n,\ r \neq 0,\ m = 0$,

$$\left.\begin{aligned}&S_n C_r\,[(\sigma_x)_{x=a}] - S_n C_r[(\sigma_x)_{x=0}] + (-1)^r\int_0^a S_n\,[(\tau_{zx})_{z=c}]\,dx \\ &\quad - \int_0^a S_n\,[(\tau_{zx})_{z=0}]\,dx - \mu N\left\{C_n C_r\,[v_{x=a}] - C_n C_r\,[v_{x=0}]\right\} \\ &\quad - \mu N\left\{(-1)^n\int_0^a C_r\,[u_{y=b}]\,dx - \int_0^a C_r\,[u_{y=0}]\,dx\right\} + \mu R\left\{S_n S_r\,[w_{x=a}]\right. \\ &\quad \left.- S_n S_r\,[w_{x=0}]\right\} - \mu(N^2+R^2)\int_0^a S_n C_r\,[u]\,dx = \int_0^a S_n C_r\,[X]\,dx\,,\end{aligned}\right\} \tag{21}$$

for $m,\ n,\ r \neq 0$,

$$\left.\begin{aligned}&(-1)^m S_n C_r\,[(\sigma_x)_{x=a}] - S_n C_r\,[(\sigma_x)_{x=0}] + (-1)^r C_m S_n\,[(\tau_{zx})_{z=c}] \\ &\quad - C_m S_n\,[(\tau_{zx})_{z=0}] - \mu N\left\{(-1)^m C_n C_r\,[v_{x=a}] - C_n C_r\,[v_{x=0}]\right\} \\ &\quad - \mu N\left\{(-1)^n C_m C_r\,[u_{y=b}] - C_m C_r\,[u_{y=0}]\right\} + \mu R\left\{(-1)^m S_n S_r\,[w_{x=a}]\right. \\ &\quad \left.- S_n S_r\,[w_{x=0}]\right\} + \lambda M\left\{(-1)^r S_m S_n\,[w_{z=c}] - S_m S_n\,[w_{z=0}]\right\} \\ &\quad - \left\{(2\mu+\lambda)M^2 + \mu N^2 + \mu R^2\right\} C_m S_n C_r\,[v] - (\mu+\lambda)MN\cdot S_m C_n C_r\,[v] \\ &\quad + (\mu+\lambda)MR\cdot S_m S_n S_r\,[w] = C_m S_n C_r\,[X]\,.\end{aligned}\right\} \tag{22}$$

Proceeding as before, we write Eq. (8) as follows:

for $m \neq n = r = 0$,

$$\left.\begin{aligned}&\int_0^c S_m\,[(\sigma_y)_{y=b}]\,dz - \int_0^c S_m\,[(\sigma_y)_{y=0}]\,dz + \int_0^b S_m\,[(\tau_{yz})_{z=c}]\,dy \\ &\quad - \int_0^b S_m\,[(\tau_{yz})_{z=0}]\,dy - \mu M\left\{\int_0^c C_m\,[u_{y=b}]\,dz - \int_0^c C_m\,[u_{y=0}]\,dz\right\} \\ &\quad - \mu M\left\{(-1)^m\int(v_{x=a})\,dA_x - \int(v_{x=0})\,dA_x\right\} - \mu\int S_m\,[v]\,dA_x = \int S_m\,[Y]\,dA_x\,,\end{aligned}\right\} \tag{23}$$

for $m,\ n \neq 0,\ r=0,$

$$\left.\begin{aligned}
&(-1)_n\int_0^c S_m\left[(\sigma_y)_{y=b}\right]dz-\int_0^c S_m\left[(\sigma_y)_{y=0}\right]dz+S_mC_n\left[(\tau_{yz})_{z=c}\right]\\
&\quad -S_mC_n\left[(\tau_{yz})_{z=0}\right]-\mu M\left\{(-1)^n\int_0^c C_m\left[u_{y=b}\right]dz-\int_0^c C_m\left[u_{y=0}\right]dz\right\}\\
&\quad -\mu M\left\{(-1)^m\int_0^c C_n\left[v_{x=a}\right]dz-\int_0^c C_n\left[v_{x=0}\right]dz\right\}-\left\{\mu M^2+(2\mu+\lambda)N^2\right\}\\
&\quad \times\int_0^c S_mC_n\left[v\right]dz-(\mu+\lambda)MN\int_0^c C_mS_n\left[u\right]dz=\int_0^c S_mC_n\left[Y\right]dz,
\end{aligned}\right\}\tag{24}$$

for $m,\ r \neq 0,\ n=0,$

$$\left.\begin{aligned}
&S_mC_r\left[(\sigma_y)_{y=b}\right]-S_mC_r\left[(\sigma_y)_{y=0}\right]+(-1)^r\int_0^b S_m\left[(\tau_{yz})_{z=c}\right]dy\\
&\quad -\int_0^b S_m\left[(\tau_{yz})_{z=0}\right]dy-\mu M\left\{C_mC_r\left[u_{y=b}\right]-C_mC_r\left[u_{y=0}\right]\right\}\\
&\quad -\mu M\left\{(-1)^m\int_0^b C_r\left[v_{x=a}\right]dy-\int_0^b C_r\left[v_{x=0}\right]dy\right\}+\mu R\left\{S_mS_r\left[w_{x=a}\right]\right.\\
&\quad \left.-S_mS_n\left[w_{x=0}\right]\right\}-\mu\left(M^2+R^2\right)\int_0^b S_mC_r\left[v\right]dy=\int_0^b S_mC_r\left[Y\right]dy,
\end{aligned}\right\}\tag{25}$$

for $m,\ n,\ r \neq 0,$

$$\left.\begin{aligned}
&(-1)^nS_mC_r\left[(\sigma_y)_{y=b}\right]-S_mC_r\left[(\sigma_y)_{y=0}\right]+(-1)^rS_mC_n\left[(\tau_{yz})_{z=c}\right]\\
&\quad -S_mC_n\left[(\tau_{yz})_{z=0}\right]-\mu M\left\{(-1)^nC_mC_r\left[u_{y=b}\right]-C_mC_r\left[u_{y=0}\right]\right\}\\
&\quad -\mu M\left\{(-1)^mC_nC_r\left[v_{x=a}\right]-C_nC_r\left[v_{x=0}\right]\right\}+\mu R\left\{(-1)^nS_mS_r\left[w_{y=b}\right]\right.\\
&\quad \left.-S_mS_r\left[w_{y=0}\right]\right\}+\lambda N\left\{(-1)^rS_mS_n\left[w_{z=c}\right]-S_mS_n\left[w_{z=0}\right]\right\}\\
&\quad -\left\{\mu M^2+(2\mu+\lambda)N^2+\mu R^2\right\}\cdot S_mC_nC_r\left[v\right]-(\mu+\lambda)MN\cdot C_mS_nC_r\left[u\right]\\
&\quad +(\mu+\lambda)NR\cdot S_mS_nS_r\left[w\right]=S_mC_nC_r\left[Y\right].
\end{aligned}\right\}\tag{26}$$

Similary, Eq. (9) yields

$$\left.\begin{aligned}
&-(2\mu+\lambda)R\left\{(-1)_rS_mS_n\left[w_{z=c}\right]-S_mS_n\left[w_{z=0}\right]\right\}-\mu M\left\{(-1)^mS_nS_r\left[w_{x=a}\right]\right.\\
&\quad \left.-S_nS_r\left[w_{x=0}\right]\right\}-\mu N\left\{(-1)^nS_mS_r\left[w_{y=b}\right]-S_mS_r\left[w_{y=0}\right]\right\}\\
&\quad -\left\{\mu M^2+\mu N^2+(2\mu+\lambda)R^2\right\}\cdot S_mS_nS_r\left[w\right]+(\mu+\lambda)RM\cdot C_mS_nC_r\left[u\right]\\
&\quad +(\mu+\lambda)NR\cdot S_mC_nC_r\left[v\right]=S_mS_nS_r\left[z\right].
\end{aligned}\right\}\tag{27}$$

Each of the three equations (22), (26) and (27) include the finite Fourier transforms of u, v, and w, Hence, the finite Fourier transforms of u, v, and w:

$$C_mS_nC_r\left[u\right],\quad S_mC_nC_r\left[u\right],\quad S_mS_nS_r\left[w\right]$$

can, with solving Eqs. (22), (26) and (27) simultaneously, be expressed by the double finite Fourier transforms connecting with the stresses and the displacements on the six side planes.

These double finite Fourier transforms are the boundary values to be determined as satisfying the given boundary conditions, and apparently the numbers of the double finite Fourier transforms are same as those of the boundary conditions.

To avoid the complexities of further evaluations, let us introduce the following notaions:

$$\left.\begin{matrix}\pi A'_{0n}\\ \pi A_{n0}\end{matrix}\right\}=\int_0^c S_n\left[(\sigma_x)_{x=a}\right]dz\mp\int_0^c S_n\left[(\sigma_x)_{x=0}\right]dz\,, \tag{28}$$

$$\left.\begin{matrix}\pi B'_{m0}\\ \pi B_{m0}\end{matrix}\right\}=\int_0^c S_m\left[(\sigma_y)_{y=b}\right]dz\mp\int_0^c S_m\left[(\sigma_y)_{y=0}\right]dz\,, \tag{29}$$

$$\left.\begin{matrix}\pi A'_{nr}\\ \pi A_{nr}\end{matrix}\right\}=S_nC_r\left[(\sigma_x)_{x=a}\right]\mp S_nC_r\left[(\sigma_x)_{x=0}\right], \tag{30}$$

$$\left.\begin{matrix}\pi B'_{mr}\\ \pi B_{mr}\end{matrix}\right\}=S_mC_r\left[(\sigma_y)_{y=b}\right]\mp S_mC_r\left[(\sigma_y)_{y=0}\right], \tag{31}$$

$$\left.\begin{matrix}\pi D'_{0n}\\ \pi D_{0n}\end{matrix}\right\}=\int_0^a S_n\left[(\tau_{zx})_{z=c}\right]dx\mp\int_0^a S_n\left[(\tau_{zx})_{z=c}\right]dx\,, \tag{32}$$

$$\left.\begin{matrix}\pi F'_{m0}\\ \pi F_{m0}\end{matrix}\right\}=\int_0^b S_m\left[(\tau_{yz})_{z=c}\right]dy\mp\int_0^b S_m\left[(\tau_{yz})_{z=0}\right]dy\,, \tag{33}$$

$$\left.\begin{matrix}\pi D'_{mn}\\ \pi D_{mn}\end{matrix}\right\}=C_mS_n\left[(\tau_{zx})_{z=c}\right]\mp C_mS_n\left[(\tau_{zx})_{z=0}\right], \tag{34}$$

$$\left.\begin{matrix}\pi F'_{mn}\\ \pi F_{mn}\end{matrix}\right\}=S_mC_n\left[(\tau_{yz})_{z=c}\right]\mp S_mC_n\left[(\tau_{yz})_{z=0}\right], \tag{35}$$

$$\left.\begin{matrix}H'_{00}\\ H_{00}\end{matrix}\right\}=\int(v_{x=a})\,dA_x\mp\int(v_{x=0})\,dA_x\,, \tag{36}$$

$$\left.\begin{matrix}J'_{00}\\ J_{00}\end{matrix}\right\}=\int(u_{y=b})\,dA_y\mp\int(u_{y=0})\,dA_y\,, \tag{37}$$

$$\left.\begin{matrix}H'_{m0}\\ H_{m0}\end{matrix}\right\}=\int_0^c C_m\left[u_{y=b}\right]dz\mp\int_0^c C_m\left[u_{y=0}\right]dz\,, \tag{38}$$

$$\left.\begin{matrix}J'_{n0}\\ J_{n0}\end{matrix}\right\}=\int_0^c C_n\left[v_{x=a}\right]dz\mp\int_0^c C_n\left[v_{x=0}\right]dz\,, \tag{39}$$

$$\left.\begin{matrix}H'_{0r}\\ H_{0r}\end{matrix}\right\}=\int_0^a C_r\left[u_{y=b}\right]dx\mp\int_0^a C_r\left[u_{y=0}\right]dx\,, \tag{40}$$

$$\left.\begin{matrix}J'_{0r}\\ J_{0r}\end{matrix}\right\}=\int_0^b C_r\left[v_{x=a}\right]dy\mp\int_0^b C_r\left[v_{x=0}\right]dy\,, \tag{41}$$

$$\left.\begin{matrix}H'_{mr}\\ H_{mr}\end{matrix}\right\}=C_mC_r\left[u_{y=b}\right]\mp C_mC_r\left[u_{y=0}\right], \tag{42}$$

$$\left.\begin{matrix}J'_{nr}\\ J_{nr}\end{matrix}\right\}=C_nC_r\left[v_{x=a}\right]\mp C_nC_r\left[v_{x=0}\right], \tag{43}$$

$$\left.\begin{array}{l} E'_{nr} \\ E_{nr} \end{array}\right\} = S_nS_r\,[w_{x=a}] \mp S_nS_r\,[w_{x=0}]\,, \tag{44}$$

$$\left.\begin{array}{l} G'_{mr} \\ G_{mr} \end{array}\right\} = S_mS_r\,[w_{y=b}] \mp S_mS_r\,[w_{y=0}]\,, \tag{45}$$

$$\left.\begin{array}{l} K'_{mn} \\ K_{mn} \end{array}\right\} = S_mS_n\,[w_{z=c}] \mp S_mS_n\,[w_{z=c}]\,. \tag{46}$$

Now, for Eqs. (19) and (23), we have

$$\left.\begin{aligned} & A_{n0}+D_{n0}-\mu NJ_{n0}-\mu N\left\{(1+(-1)^n)\,H_{00}+(1-(-1)^n)\,H'_{00}\right\} \\ & \quad -\mu N^2\int S_n\,[u]\,dA_y-\mu N^2\int S_n\,[u]\,dA_y=\int S_n\,[X]\,dA_y\,, \\ & B_{m0}+F_{m0}-\mu MH_{m0}-\mu M\left\{(1+(-1)^m)\,J_{00}+(1-(-1)^m)\,J'_{00}\right\} \\ & \quad -\mu M^2\int S_m\,[v]\,dA_x-\mu M^2\int S_m\,[v]\,dA_x=\int S_n\,[Y]\,dA_x\,, \end{aligned}\right\} \tag{47}$$

for Eqs. (21) and (25),

$$\left.\begin{aligned} & A_{nr}+D_{0n}-\mu NJ_{nr}-\mu N\left\{(1+(-1)^n)\,H_{0r}+(1-(-1)^n)H'_{0r}\right\} \\ & \quad +\mu RE_{nr}-\int_0^a S_nC_r\,[X]\,dx=\mu\,(N^2+R^2)\int_0^a S_nC_r\,[u]\,dx\,, \\ & B_{mr}+F_{m0}-\mu MH_{mr}-\mu M\left\{(1+(-1)^n)\,J_{0r}+(1-(-1)^n)J'_{0r}\right\} \\ & \quad +\mu RG_{mr}-\int_0^b S_mC_r\,[Y]\,dy=\mu\,(M^2+R^2)\int_0^b S_mC_r\,[v]\,dy\,, \end{aligned}\right\} \tag{48}$$

for Eqs. (20) and (24),

$$\left.\begin{aligned} & (1+(-1)^m)\,A_{n0}+(1-(-1)^m)\,A'_{0n}+D_{mn}-\mu N\left\{(1+(-1)^m)\,J_{n0}\right. \\ & \quad \left.+(1-(-1)^m)\,J'_{n0}\right\}-\mu N\left\{(1+(-1)^n)\,H_{m0}+(1-(-1)^n)\,H'_{m0}\right\} \\ & \quad -\int_0^c C_mS_n\,[X]\,dx=\left\{(2\mu+\lambda)\,M^2+\mu N^2\right\}\int_0^c C_mS_n\,[u]\,dz \\ & \quad +MN(\mu+\lambda)\int_0^c S_mC_n\,[v]\,dz\,, \\ & (1+(-1)^n)\,B_{m0}+(1-(-1)^n)\,B'_{m0}+F_{mn}-\mu M\left\{(1+(-1)^n)\,H_{m0}\right. \\ & \quad \left.+(1-(-1)^n)\,H'_{m0}\right\}-\mu M\left\{(1+(-1)^m)\,J_{n0}+(1-(-1)^m)\,J'_{n0}\right\} \\ & \quad -\int_0^c S_mC_n\,[Y]\,dz=\left\{(2\mu+\lambda)\,N^2+\mu M^2\right\}\int_0^c S_mC_n\,[v]\,dz \\ & \quad +MN\,(\mu+\lambda)\int_0^c C_mS_n\,[u]\,dz\,, \end{aligned}\right\} \tag{49}$$

from which

$$\left.\begin{aligned}&\int_0^c C_mS_n[u]\,dz=\frac{1}{\mu}\left\{\frac{1}{M^2+N^2}-\frac{\mu+\lambda}{2\mu+\lambda}\frac{M^2}{(M^2+N^2)^2}\right\}\Big\{(1+(-1)^m)A_{n0}\\&\quad+(1-(-1)^m)A'_{n0}-\int_0^c C_mS_n[X]\,dz\Big\}-\frac{1}{\mu}\frac{\mu+\lambda}{2\mu+\lambda}\frac{MN}{(M^2+N^2)^2}\\&\quad\times\Big\{(1+(-1)^n)B_{m0}+(1-(-1)^n)B'_{m0}-\int_0^c S_mC_n[Y]\,dz\Big\},\end{aligned}\right\}\tag{50}$$

$$\left.\begin{aligned}&\int_0^c S_mC_n[v]\,dz=\frac{1}{\mu}\left\{\frac{1}{M^2+N^2}-\frac{\mu+\lambda}{2\mu+\lambda}\frac{N^2}{(M^2+N^2)}\right\}\Big\{(1+(-1)^n)B_{m0}\\&\quad+(1-(-1)^n)B'_{m0}-\int_0^c S_mC_n[Y]\,dz\Big\}-\frac{1}{\mu}\frac{\mu+\lambda}{2\mu+\lambda}\frac{MN}{(M^2+N^2)^2}\\&\quad\times\Big\{(1+(-1)^m)A_{n0}+(1-(-1)^m)A'_{n0}-\int_0^c C_mS_n[X]\,dz\Big\},\end{aligned}\right\}\tag{51}$$

for Eqs. (22), (26) and (27), we have

$$\left.\begin{aligned}&(1+(-1)^m)A_{nr}+(1-(-1)^m)A'_{nr}+(1+(-1)^r)D_{mn}+(1-(-1)^r)D'_{mn}\\&\quad-\mu N\{(1+(-1)^m)J_{nr}+(1-(-1)^m)J'_{nr}\}+\mu R\{(1+(-1)^m)E_{nr}\\&\quad+(1-(-1)^m)E'_{nr}\}-\mu N\{(1+(-1)^n)H_{mr}+(1-(-1)^n)H'_{mr}\}\\&\quad+\lambda M\{(1+(-1)^r)K_{mn}+(1-(-1)^r)K'_{mn}\}-C_mS_nC_r[X]\\&\quad=\{(2\mu+\lambda)M^2+\mu N^2+\mu R^2\}\times C_mS_nC_r[u]+(\mu+\lambda)MN\cdot S_mC_nC_r[v]\\&\quad-(\mu+\lambda)MR\cdot S_mS_nS_r[w],\end{aligned}\right\}\tag{52}$$

$$\left.\begin{aligned}&(1+(-1)^n)B_{nr}+(1-(-1)^n)B'_{nr}+(1+(-1)^r)F_{mn}-(1-(-1)^r)F'_{mn}\\&\quad-\mu M\{(1+(-1)^n)H_{mr}+(1-(-1)^n)H'_{mr}\}+\mu R\{(1+(-1)^n)G_{mr}\\&\quad+(1-(-1)^n)G'_{mr}\}-\mu M\{(1+(-1)^m)J_{nr}+(1-(-1)^m)J'_{nr}\}\\&\quad+\lambda N\{(1+(-1)^r)K_{mn}+(1-(-1)^r)K'_{mn}\}-S_mC_nC_r[Y]\\&\quad=\{(2\mu+\lambda)N^2+\mu M^2+\mu R^2\}\times S_mC_nC_r[v]+(\mu+\lambda)MN\times C_mS_nC_r[u]\\&\quad-(\mu+\lambda)NR\cdot S_mS_nS_r[w],\end{aligned}\right\}\tag{53}$$

$$\left.\begin{aligned}&-(2\mu+\lambda)R\{(1+(-1)^r)K_{mn}-(1-(-1)^r)K'_{mn}\}-\mu M\{(1+(-1)^m)E_{nr}\\&\quad+(1-(-1)^m)E'_{nr}\}-\mu M\{(1+(-1)^n)G_{mr}+(1-(-1)^n)G'_{mr}\}\\&\quad-S_mS_nS_r[Z]=\{(2\mu+\lambda)R^2+\mu M^2+\mu N^2\}\times S_mS_nS_r[w]\\&\quad-(\mu+\lambda)R\cdot M\times C_mS_nC_r[u]-(\mu+\lambda)NR\times S_mC_nC_r[v],\end{aligned}\right\}\tag{54}$$

from which we find that

$$\begin{aligned}&C_mS_nC_r[u]=\left\{\frac{\lambda}{2\mu+\lambda}\frac{M}{T}-\frac{2(\mu+\lambda)}{2\mu+\lambda}\frac{MR^2}{T^2}\right\}\Big\{(1+(-1)^r)K_{mn}\\&\quad+(1-(-1)^r)K'_{mn}\Big\}+\frac{2(\mu+\lambda)}{2\mu+\lambda}\frac{MNR}{T^2}\Big\{(1+(-1)^n)G_{mr}+(1-(-1)^n)G'_{mr}\Big\}\\&\quad-\left\{\frac{R}{T}-\frac{2(\mu+\lambda)}{2\mu+\lambda}\frac{M^2R}{T^2}\right\}\Big\{(1+(-1)^m)E_{nr}+(1-(-1)^m)E'_{nr}\Big\}\end{aligned}$$

$$\left.\begin{aligned}
&-\frac{\mu+\lambda}{\mu(2\mu+\lambda)}\frac{MN}{T^2}\times\Big\{(1+(-1)^n)B_{mr}+(1-(-1)^n)B'_{mr}+(1+(-1)^r)F_{mn}\\
&+(1-(-1)^r)F'_{mn}-S_mC_nC_r[Y]\Big\}+\Big\{\frac{1}{\mu T}-\frac{\mu+\lambda}{\mu(2\mu+\lambda)}\frac{M^2}{T^2}\Big\}\Big\{(1+(-1)^m)\\
&\times A_{nr}+(1-(-1)^m)A'_{nr}+(1+(-1)^r)D_{mn}+(1-(-1)^r)D'_{mn}-C_mS_nC_r[X]\Big\}\\
&-\Big\{\frac{N}{T}-\frac{2(\mu+\lambda)}{2\mu+\lambda}\frac{M^2N}{T^2}\Big\}\Big\{(1+(-1)^n)H_{mr}+(1-(-1)^n)H'_{mr}\\
&+(1+(-1)^m)J_{nr}+(1-(-1)^m)J'_{nr}\Big\}+\frac{\lambda}{2\mu+\lambda}\frac{MR}{T^2}S_mS_nS_c[Z],
\end{aligned}\right\}\tag{55}$$

$$\left.\begin{aligned}
S_mC_nC_r[v]=&\Big\{\frac{\lambda}{2\mu+\lambda}\frac{N}{T}-\frac{2(\mu+\lambda)}{2\mu+\lambda}\frac{NR^2}{T^2}\Big\}\Big\{(1+(-1)^r)K_{mn}\\
&+(1-(-1)^r)K'_{mn}\Big\}+\frac{2(\mu+\lambda)}{2\mu+\lambda}\frac{MNR}{T^2}\Big\{(1+(-1)^m)E_{nr}+(1-(-1)^m)E'_{nr}\Big\}\\
&-\Big\{\frac{R}{T}-\frac{2(\mu+\lambda)}{2\mu+\lambda}\frac{N^2R}{T^2}\Big\}\times\Big\{(1+(-1)^n)G_{mr}+(1-(-1)^n)G'_{mr}\Big\}\\
&-\frac{\mu+\lambda}{\mu(2\mu+\lambda)}\frac{MN}{T^2}\Big\{(1+(-1)^m)A_{nr}+(1-(-1)^m)A'_{nr}+(1+(-1)^r)D_{mn}\\
&+(1-(-1)^r)D'_{mn}-C_mS_nC_r[X]\Big\}+\Big\{\frac{1}{\mu T}-\frac{2(\mu+\lambda)}{\mu(2\mu+\lambda)}\frac{N^2}{T^2}\Big\}\\
&\times\Big\{(1+(-1)^n)B_{mr}+(1-(-1)^n)B'_{mr}+(1+(-1)^r)F_{mn}+(1+(-1)^r)F'_{mn}\\
&-S_mC_nC_r[Y]\Big\}-\Big\{\frac{M}{T}-\frac{2(\mu+\lambda)}{2\mu+\lambda}\frac{MN^2}{T^2}\Big\}\Big\{(1+(-1)^n)H_{mr}\\
&+(1-(-1)^n)H'_{mr}+(1+(-1)^m)J_{nr}+(1-(-1)^m)J'_{nr}\Big\}\\
&+\frac{\lambda}{2\mu+\lambda}\frac{NR}{T^2}S_mS_nS_c[Z],
\end{aligned}\right\}\tag{56}$$

$$\left.\begin{aligned}
S_mS_nS_r[w]=&-\Big\{\frac{R}{T}+\frac{2(\mu+\lambda)}{2\mu+\lambda}\frac{R(M^2+N^2)}{T^2}\Big\}\Big\{(1+(-1)^r)K_{mn}\\
&+(1-(-1)^r)K'_{mn}\Big\}+\Big\{\frac{M}{T}-\frac{2(\mu+\lambda)}{2\mu+\lambda}\frac{MR^2}{T^2}\Big\}\Big\{(1+(-1)^m)E_{nr}\\
&+(1-(-1)^m)E'_{nr}\Big\}+\Big\{\frac{N}{T}-\frac{2(\mu+\lambda)}{2\mu+\lambda}\frac{NR^2}{T^2}\Big\}\Big\{(1+(-1)^n)G_{mr}\\
&+(1-(-1)^n)G'_{mr}\Big\}+\frac{\mu+\lambda}{\mu(2\mu+\lambda)}\frac{RM}{T^2}\Big[(1+(-1)^m)A_{nr}+(1-(-1)^m)A'_{nr}\\
&+(1+(-1)^r)D_{mn}+(1-(-1)^r)D'_{mn}-2\mu N\Big\{(1+(-1)^m)J_{nr}\\
&+(1-(-1)^m)J'_{nr}\Big\}-C_mS_nC_r[X]\Big]+\frac{\mu+\lambda}{\mu(2\mu+\lambda)}\frac{NR}{T^2}\Big[(1+(-1)^n)B_{mr}
\end{aligned}\right\}\tag{57}$$

$$\left.\begin{aligned}&+(1-(-1)^n)B'_{mr}+(1+(-1)^r)F_{mn}+(1-(-1)^r)F'_{mn}-2\mu M\\&\times\Big\{(1+(-1)^n)H_{mr}+(1-(-1)^n)H'_{mr}\Big\}-S_mC_nC_r[Y]\Big]\\&-\left\{\frac{1}{T}-\frac{\mu+\lambda}{\mu(2\mu+\lambda)}\frac{R^2}{T^2}\right\}\times S_mS^nS_r[Z]\end{aligned}\right\}$$

where

$$T=M^2+N^2+R^2=\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2+\left(\frac{r\pi}{c}\right)^2.$$

With the aid of the following formulas:

$$\left.\begin{aligned}&\sum_m\begin{matrix}(1-(-1)^m)\\(1+(-1)^m)\end{matrix}\frac{M}{T}\sin Mx=\frac{a}{2}\begin{cases}Q^{(1)}(\alpha_{nr}\xi),\\Q^{(2)}(\alpha_{nr}\xi),\end{cases}\\&\sum_m\begin{matrix}(1-(-1)^m)\\(1+(-1)^m)\end{matrix}\frac{M}{T^2}\sin Mx=\frac{a^3}{4\pi^2\alpha_{nr}^2}\begin{cases}P^{(1)}(\alpha_{nr}\xi),\\P^{(2)}(\alpha_{nr}\xi),\end{cases}\\&\begin{matrix}Q^{(1)}(\alpha_{nr}\xi)\\Q^{(2)}(\alpha_{nr}\xi)\end{matrix}\Big\}=\frac{ch\pi\alpha_{nr}(1-\xi)\pm ch\pi\alpha_{nr}\xi}{ch\pi\alpha_{nr}\pm1},\\&\begin{matrix}P^{(1)}(\alpha_{nr}\xi)\\P^{(2)}(\alpha_{nr}\xi)\end{matrix}\Big\}=\frac{\pi\alpha_{nr}\{\xi sh\pi\alpha_{nr}(1-\xi)\pm(1-\xi)sh\pi\alpha_{nr}\}}{ch\pi\alpha_{nr}\pm1},\end{aligned}\right\}\quad(58)$$

$$M=\frac{m\pi}{a},\quad T=\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2+\left(\frac{r\pi}{c}\right)^2,\quad \alpha_{nr}^2=\frac{a^2}{b^2}n^2+\frac{a^2}{c^2}r^2,\quad \xi=\frac{x}{a},$$

$m=1,\ 2,\ 3,\ 4,\cdots\cdots,$

$$\left.\begin{aligned}&\sum_n\begin{matrix}(1-(-1)^n)\\(1+(-1)^n)\end{matrix}\frac{1}{T}\cos Mx=\frac{a^2}{2\pi\alpha_{nr}}\begin{cases}\phi^{(1)}(\alpha_{nr}\xi),\\\phi^{(2)}(\pi_{nr}\xi)-\dfrac{2}{\pi\alpha_{nr}},\end{cases}\\&\sum_n\begin{matrix}(1-(-1)^n)\\(1+(-1)^n)\end{matrix}\frac{M^2}{T^2}\cos Mx=\frac{a^2}{4\pi\alpha_{nr}}\begin{cases}\phi^{(1)}(\alpha_{nr}\xi)-\Psi^{(1)}(\alpha_{nr}\xi),\\\phi^{(2)}(\alpha_{nr}\xi)-\Psi^{(2)}(\alpha_{nr}\xi),\end{cases}\\&\sum_n\begin{matrix}(1-(-1)^n)\\(1+(-1)^n)\end{matrix}\frac{1}{T^2}\cos Mx=\frac{a^4}{4\pi^3\alpha_{nr}^3}\begin{cases}\phi^{(1)}(\alpha_{nr}\xi)+\Psi^{(1)}(\alpha_{nr}\xi),\\\phi^{(2)}(\alpha_{nr}\xi)+\Psi^{(2)}(\alpha_{nr}\xi)-\dfrac{2}{\pi\alpha_{nr}},\end{cases}\\&\begin{matrix}\phi^{(1)}(\alpha_{nr}\xi)\\\phi^{(2)}(\alpha_{nr}\xi)\end{matrix}\Big\}=\frac{sh\pi\alpha_{nr}(1-\xi)\mp sh\pi\alpha_{nr}\xi}{ch\pi\alpha_{nr}\pm1},\\&\begin{matrix}\Psi^{(1)}(\alpha_{nr}\xi)\\\Psi^{(2)}(\alpha_{nr}\xi)\end{matrix}\Big\}=\frac{\pi\alpha_{nr}\{\xi ch\pi\alpha_{nr}(1-\xi)\mp(1-\xi)ch\pi\alpha_{nr}\xi\}}{ch\pi\alpha_{nr}\pm1},\end{aligned}\right\}\quad(59)$$

$m=1,\ 2,\ 3,\ 4,\cdots\cdots,$

$$\left.\begin{aligned}&\sum_n\begin{matrix}(1-(-1)^n)\\(1+(-1)^n)\end{matrix}\frac{N}{T}\sin Ny=\frac{b}{2}\begin{cases}Q^{(1)}(\beta_{mr}\eta),\\Q^{(2)}(\beta_{mr}\eta),\end{cases}\\&\sum_n\begin{matrix}(1-(-1)^n)\\(1+(-1)^n)\end{matrix}\frac{N}{T^2}\sin Ny=\frac{b^3}{4\pi^2\beta_{mr}}\begin{cases}P^{(1)}(\beta_{mr}\eta),\\P^{(2)}(\beta_{mr}\eta),\end{cases}\end{aligned}\right\}\quad(60)$$

$$\left.\begin{array}{l}
\left.\begin{array}{l} Q^{(1)}(\beta_{mr}\eta) \\ Q^{(2)}(\beta_{mr}\eta) \end{array}\right\} = \dfrac{ch\pi\beta_{mr}(1-\eta) \pm ch\pi\beta_{mr}\eta}{ch\pi\beta_{mr} \pm 1}\,, \\
\left.\begin{array}{l} P^{(1)}(\beta_{mr}\eta) \\ P^{(2)}(\beta_{mr}\eta) \end{array}\right\} = \dfrac{\pi\beta_{mr}\{\eta sh\pi\beta_{mr}(1-\eta) \pm (1-\eta) sh\pi\beta_{mr}\eta\}}{ch\pi\beta_{mr} \pm 1}\,,
\end{array}\right\}$$

$$\beta_{mr}^2 = \frac{b^2}{a^2}m^2 + \frac{b^2}{c^2}r^2, \quad \eta = \frac{y}{b}, \quad N = \frac{n\pi}{b},$$

$$n = 1,\ 2,\ 3,\ 4,\ \cdots\cdots,$$

$$\left.\begin{array}{l}
\sum\limits_n \begin{array}{c}(1-(-1)^n) \\ (1+(-1)^n)\end{array} \dfrac{1}{T}\cos Ny = \dfrac{a^2}{2\pi\beta_{mr}} \begin{cases} \phi^{(1)}(\beta_{mr}\eta), \\ \phi^{(2)}(\beta_{mr}\eta) - \dfrac{2}{\pi\beta_{mr}}, \end{cases} \\
\sum\limits_n \begin{array}{c}(1-(-1)^n) \\ (1+(-1)^n)\end{array} \dfrac{N^2}{T^2}\cos Ny = \dfrac{a^2}{4\pi\beta_{mr}} \begin{cases} \phi^{(1)}(\beta_{mr}\eta) - \Psi^{(1)}(\beta_{mr}\eta), \\ \phi^{(2)}(\beta_{mr}\eta) - \Psi^{(2)}(\beta_{mr}\eta), \end{cases} \\
\sum\limits_n \begin{array}{c}(1-(-1)^n) \\ (1+(-1)^n)\end{array} \dfrac{1}{T^2}\cos Ny = \dfrac{a^4}{4\pi^3\beta_{mr}^3} \begin{cases} \phi^{(1)}(\beta_{mr}\eta) + \Psi^{(1)}(\beta_{mr}\eta), \\ \phi^{(2)}(\beta_{mr}\eta) + \Psi^{(2)}(\beta_{mr}\eta) - \dfrac{2}{\pi\beta_{mr}}, \end{cases} \\
\left.\begin{array}{l} \phi^{(1)}(\beta_{mr}\eta) \\ \phi^{(2)}(\beta_{mr}\eta) \end{array}\right\} = \dfrac{sh\pi\beta_{mr}(1-\eta) \mp sh\pi\beta_{mr}\eta}{ch\pi\beta_{mr} \pm 1}\,, \\
\left.\begin{array}{l} \Psi^{(1)}(\beta_{mr}\eta) \\ \Psi^{(2)}(\beta_{mr}\eta) \end{array}\right\} = \dfrac{\pi\beta_{mr}\{(1-\eta) ch\pi\beta_{m}\eta \mp \eta ch\pi\beta_{mr}(1-\eta)\}}{ch\pi\beta_{mr} \pm 1}\,,
\end{array}\right\} \quad (61)$$

$$n = 1,\ 2,\ 3,\ 4,\ \cdots\cdots,$$

$$\left.\begin{array}{l}
\sum\limits_r \begin{array}{c}(1-(-1)^r) \\ (1+(-1)^r)\end{array} \dfrac{R}{T}\sin Rz = \dfrac{c}{2} \begin{cases} Q^{(1)}(\gamma_{mn}\zeta), \\ Q^{(2)}(\gamma_{mn}\zeta), \end{cases} \\
\sum\limits_r \begin{array}{c}(1-(-1)^r) \\ (1+(-1)^r)\end{array} \dfrac{R}{T^2}\sin Rz = \dfrac{c^3}{4\pi^2\gamma_{mn}^2} \begin{cases} P^{(1)}(\gamma_{mn}\zeta), \\ P^{(2)}(\gamma_{mn}\zeta), \end{cases} \\
\left.\begin{array}{l} Q^{(1)}(\gamma_{mn}\zeta) \\ Q^{(2)}(\gamma_{mn}\zeta) \end{array}\right\} = \dfrac{ch\pi\gamma_{mn}(1-\zeta) \mp ch\pi\gamma_{mn}\zeta}{ch\pi\gamma_{mn} \pm 1}\,, \\
\left.\begin{array}{l} P^{(1)}(\gamma_{mn}\zeta) \\ P^{(2)}(\gamma_{mn}\zeta) \end{array}\right\} = \dfrac{\pi\gamma_{mn}\{(1-\zeta) sh\pi\gamma_{mn}\zeta \pm \zeta sh\pi\gamma_{mn}(1-\zeta)\}}{ch\pi\gamma_{mn} \pm 1}\,,
\end{array}\right\} \quad (62)$$

$$\gamma_{mn}^2 = \frac{c^2}{a^2}m^2 + \frac{c^2}{b^2}n^2, \quad \zeta = \frac{z}{c}, \quad R = \frac{r\pi}{c},$$

$$r = 1,\ 2,\ 3,\ 4,\ \cdots\cdots,$$

$$\left.\begin{array}{l}
\sum\limits_r \begin{array}{c}(1-(-1)^r) \\ (1+(-1)^r)\end{array} \dfrac{1}{T}\cos Rz = \dfrac{c^2}{2\pi\gamma_{mn}} \begin{cases} \phi^{(1)}(\gamma_{mn}\zeta), \\ \phi^{(1)}(\gamma_{mn}\zeta) - \dfrac{2}{\pi\gamma_{mn}}, \end{cases} \\
\sum\limits_r \begin{array}{c}(1-(-1)^r) \\ (1+(-1)^r)\end{array} \dfrac{R^2}{T^2}\cos Rz = \dfrac{c^2}{4\pi\gamma_{mn}} \begin{cases} \phi^{(1)}(\gamma_{mn}\zeta) - \Psi^{(1)}(\gamma_{mn}\zeta), \\ \phi^{(2)}(\gamma_{mn}\zeta) - \Psi^{(2)}(\gamma_{mn}\zeta), \end{cases} \\
\sum\limits_r \begin{array}{c}(1-(-1)^r) \\ (1+(-1)^r)\end{array} \dfrac{1}{T^2}\cos Rz = \dfrac{c^3}{4\pi^3\gamma_{mn}^3} \begin{cases} \phi^{(1)}(\gamma_{mn}\zeta) + \Psi^{(1)}(\gamma_{mn}\zeta) \\ \phi^{(2)}(\gamma_{mn}\zeta) + \Psi^{(2)}(\gamma_{mn}\zeta) - \dfrac{2}{\pi\gamma_{mn}}, \end{cases}
\end{array}\right\} \quad (63)$$

$$\left.\begin{aligned}
&\left.\begin{aligned}\phi^{(1)}(\gamma_{mn}\zeta)\\ \phi^{(2)}(\gamma_{mn}\zeta)\end{aligned}\right\} = \frac{sh\pi\gamma_{mn}(1-\zeta)\mp sh\pi\gamma_{mn}\zeta}{ch\pi\gamma_{mn}\pm 1},\\
&\left.\begin{aligned}\Psi^{(1)}(\gamma_{mn}\zeta)\\ \Psi^{(2)}(\gamma_{mn}\zeta)\end{aligned}\right\} = \frac{\pi\gamma_{mn}\{\zeta ch\pi\gamma_{mn}(1-\zeta)\mp(1-\zeta)ch\pi\gamma_{mn}\zeta\}}{ch\pi\gamma_{mn}\pm 1},\\
&\quad r = 1,\ 2,\ 3,\ 4,\ \cdots\cdots,
\end{aligned}\right|$$

the inversion theorem stated by Eqs. (17), (16) and (15), yield the components of the displacement without the body forces X, Y, and Z, as follows:

$$\left.\begin{aligned}
u = {}& \sum_m\sum_n \frac{c^2m}{2a\gamma_{mn}}\Big[K_{mn}\Big\{-\frac{\mu}{2\mu+\lambda}\phi^{(1)}(\gamma_{mn}\zeta)+\frac{\mu+\lambda}{2\mu+\lambda}\Psi^{(1)}(\gamma_{mn}\zeta)\Big\}\\
&+K'_{mn}\Big\{-\frac{\mu}{2\mu+\lambda}\phi^{(2)}(\gamma_{mn}\zeta)+\frac{\mu+\lambda}{2\mu+\lambda}\Psi^{2}(\gamma_{mn}\zeta)\Big\}\Big]\times\cos\frac{m\pi}{a}x\\
&\times\sin\frac{n\pi}{b}y+\sum_m\sum_n\frac{b^3mr}{2ca\beta_{mr}^2}\,\frac{\mu+\lambda}{2\mu+\lambda}\Big\{G_{mr}P^{(1)}(\beta_{mr}\eta)+G'_{mr}P^{(2)}(\beta_{mr}\eta)\Big\}\\
&\times\cos\frac{m\pi}{a}x\times\cos\frac{r\pi}{c}z-\sum_n\sum_r\frac{ra^2}{2ca_{nr}}\Big[E_{nr}\Big\{\frac{\mu}{2\mu+\lambda}\phi^{(1)}(\alpha_{nr}\xi)\\
&+\frac{\pi+\lambda}{2\mu+\lambda}\Psi^{(1)}(\alpha_{nr}\xi)\Big\}+E'_{nr}\Big\{\frac{\mu}{2\mu+\lambda}\phi^{(2)}(\alpha_{nr}\xi)+\frac{\mu+\lambda}{2\mu+\lambda}\Psi^{(2)}(\alpha_{nr}\xi)\Big\}\Big]\cdot\\
&\sin\frac{n\pi}{b}y\cos\frac{r\pi}{c}z+\sum_m\sum_r\frac{b}{2}\Big[H_{mr}\Big\{Q^{(1)}(\beta_{mr}\eta)-\frac{\mu+\lambda}{2\mu+\lambda}\,\frac{1}{\beta_{mr}^2}\Big(\frac{mb}{a}\Big)^2\\
&\times P^{(1)}(\beta_{mr}\eta)\Big\}+H'_{mr}\Big\{Q^{(2)}(\beta_{mr}\eta)-\frac{\mu+\lambda}{2\mu+\lambda}\,\frac{1}{\beta_{mr}^2}\Big(\frac{mb}{a}\Big)^2\cdot P^{(2)}(\beta_{mr}\eta)\Big\}\Big]\cdot\\
&\cos\frac{m\pi}{a}x\cdot\cos\frac{r\pi}{c}z-\sum_n\sum_r\frac{a^2n}{2b\alpha_{nr}}\Big[J_{nr}\Big\{\frac{\mu}{2\mu+\lambda}\phi^{(1)}(\alpha_{nr}\xi)+\frac{\mu+\lambda}{2\mu+\lambda}\\
&\Psi_{(1)}(\alpha_{nr}\xi)\Big\}+J'_{nr}\Big\{\frac{\mu}{2\mu+\lambda}P^{(2)}(\alpha_{nr}\xi)+\frac{\mu+\lambda}{2\mu+\lambda}\Psi^{(2)}(\alpha_{nr}\xi)\Big\}\Big]\cdot\sin\frac{n\pi}{b}y\cdot\\
&\cos\frac{r\pi}{c}z-\sum_m\sum_r\frac{b^3m}{4a\beta_{mr}^2}\,\frac{\mu+\lambda}{\mu(2\mu+\lambda)}\Big\{B_{mr}P^{(1)}(\beta_{mr}\eta)+B'_{mr}P^{(2)}(\beta_{mr}\eta)\Big\}\\
&\times\cos\frac{m\pi}{a}x\cdot\cos\frac{r\pi}{c}z-\sum_m\sum_n\frac{c^4mn}{4ab(\gamma_{mn})^3}\,\frac{\mu+\lambda}{\mu(2\mu+\lambda)}\Big[F_{mn}\Big\{\phi^{(1)}(\gamma_{mn}\zeta)\\
&+\Psi^{(1)}(\gamma_{mn}\zeta)\Big\}+F'_{mn}\Big\{\phi^{(2)}(\gamma_{mn}\zeta)+\Psi^{(2)}(\gamma_{mn}\zeta)\Big\}\Big]\cdot\cos\frac{m\pi}{a}x\cdot\sin\frac{n\pi}{b}y\\
&+\sum_n\sum_r\frac{a^2}{4\alpha_{nr}}\Big[A_{nr}\Big\{\frac{3\mu+\lambda}{\mu(2\mu+\lambda)}\phi^{(1)}(\alpha_{nr}\xi)+\frac{\mu+\lambda}{\mu(2\mu+\lambda)}\Psi^{(1)}(\alpha_{nr}\xi)\Big\}\\
&+A'_{nr}\Big\{\frac{3\mu+\lambda}{u(2\mu+\lambda)}\phi^{(2)}(\alpha_{nr}\xi)+\frac{\mu+\lambda}{\mu(2\mu+\lambda)}\Psi^{(2)}(\alpha_{nr}\xi)\Big\}\Big]\sin\frac{n\pi}{b}y\cdot\cos\frac{r\pi}{c}z\\
&+\sum_m\sum_n\frac{c^2}{4\gamma_{mn}\mu}\Big[D_{mn}\Big\{\Big(2-\frac{\mu+\lambda}{2\mu+\lambda}\,\frac{c^2m^2}{a^2\gamma_{mn}^2}\Big)\times\phi^{(1)}(\gamma_{mn}\zeta)-\frac{\mu+\lambda}{2\mu+\lambda}\\
&\times\frac{c^2m^2}{a^2\gamma_{mn}^2}\Psi^{(1)}(\gamma_{mn}\zeta)\Big\}+D'_{mn}\Big\{\Big(2-\frac{\mu+\lambda}{2\mu+\lambda}\,\frac{c^2m^2}{a^2\gamma_{mn}^2}\Big)\cdot\phi^{(2)}(\gamma_{mn}\zeta)-\frac{\mu+\lambda}{2\mu+\lambda}
\end{aligned}\right\}\quad(64)$$

$$
\begin{aligned}
&\times \frac{c^2m^2}{a^2\gamma_{mn}^2}\Psi^{(2)}(\gamma_{mn}\zeta)\Big\}\Big]\cos\frac{m\pi}{a}x\cdot\sin\frac{n\pi}{b}y+\frac{a^2}{8\mu}\sum_n\frac{b}{an}\Big[A_{n0}\Big\{\frac{3\mu+\lambda}{2\mu+\lambda}\\
&\times\phi^{(1)}(\alpha_{n0}\xi)+\frac{\mu+\lambda}{2\mu+\lambda}\Psi^{(1)}(d_{n0}\xi)\Big\}+A'_{n0}\Big\{\frac{3\mu+\lambda}{2\mu+\lambda}\times\phi^{(2)}(\alpha_{n0}\xi)+\frac{\mu+\lambda}{2\mu+\lambda}\\
&\times\Psi^{(2)}(\alpha_{n0}\xi)\Big\}\Big]\sin\frac{n\pi}{b}y+\frac{b}{4}\sum_m\Big[H_{m0}\Big\{Q^{(1)}(\beta_{m0}\eta)-\frac{\mu+\lambda}{2\mu+\lambda}P^{(1)}(\beta_{m0}\eta)\Big\}\\
&+H'_{m0}\Big\{Q^{(2)}(\beta_{m0}\eta)-\frac{\mu+\lambda}{2\mu+\lambda}P^{(2)}(\beta_{m0}\eta)\Big\}\Big]\cos\frac{m\pi}{a}x+\frac{a}{4}\sum_n\Big[J_{n0}\Big\{\frac{\mu}{2\mu+\lambda}\\
&\times\phi^{(1)}(\alpha_{n0}\xi)+\frac{\mu+\lambda}{2\mu+\lambda}\Psi^{(1)}(\alpha_{n0}\xi)\Big\}+J'_{n0}\Big\{\frac{\mu}{2\mu+\lambda}\phi^{(2)}(\alpha_{n0}\xi)+\frac{\mu+\lambda}{2\mu+\lambda}\\
&\times\Psi^{(2)}(\alpha_{n0}\xi)\Big\}\Big]\sin\frac{n\pi}{b}y-\sum_m\frac{ab}{8m}\frac{1}{2\mu+\lambda}\Big\{B_{m0}\times P^{(1)}(\beta_{m0}\eta)+B'_{m0}\\
&\times P^{(2)}(\beta_{m0}\eta)\Big\}\cos\frac{m\pi}{a}x+\sum_n\frac{bc}{4n}\Big\{D_{0n}\phi^{(1)}(\gamma_{0n}\zeta)+D'_{0n}\phi^{(2)}(\gamma_{0n}\zeta)\Big\}\sin\frac{n\pi}{b}y\\
&-\frac{b}{4}\sum_r\Big\{J_{0r}Q^{(1)}(\beta_{0r}\eta)+J'_{0r}Q^{(2)}(\beta_{0r}\eta)\Big\}\cos\frac{r\pi}{c}z-\frac{b}{8}\Big\{J_{00}(1-2\eta)+J'_{00}\Big\}\\
&\quad m,\ n,\ r=1,\ 2,\ 3,\ 4,\ \cdots\cdots,
\end{aligned}
$$

$$
\begin{aligned}
v=&-\sum_m\sum_n\frac{c^2n}{2b\gamma_{mn}}\Big[K_{mn}\Big\{\frac{\mu}{2\mu+\lambda}\phi^{(1)}(\gamma_{mn}\zeta)-\frac{\mu+\lambda}{2\mu+\lambda}\Psi^{(1)}(\gamma_{mn}\zeta)\Big\}\\
&+K'_{mn}\Big\{\frac{\mu}{2\mu+\lambda}\phi^{(2)}(\gamma_{mn}\zeta)-\frac{\mu+\lambda}{2\mu+\lambda}\Psi^{(2)}(\gamma_{mn}\zeta)\Big\}\Big]\sin\frac{m\pi}{a}x\cos\frac{n\pi}{b}y\\
&+\sum_n\sum_r\frac{a^3nr}{2cb\alpha_{nr}^2}\frac{\mu+\lambda}{2\mu+\lambda}\Big\{E_{nr}P^{(1)}(\alpha_{nr}\xi)+E'_{nr}P^{(2)}(\alpha_{nr}\xi)\Big\}\cos\frac{n\pi}{b}y\cos\frac{r\pi}{c}z\\
&-\sum_n\sum_r\frac{a^3n}{4b\alpha_{nr}^2}\frac{\mu+\lambda}{\mu(2\mu+\lambda)}\Big\{A_{nr}P^{(1)}(\alpha_{nr}\xi)+A'_{nr}P^{(2)}(\alpha_{nr}\xi)\Big\}\cos\frac{n\pi}{b}y\cdot\cos\\
&\times\frac{r\pi}{c}z-\sum_m\sum_r\frac{rb^2}{2c\beta_{mr}}\Big[G_{mr}\Big\{\frac{\mu}{2\mu+\lambda}\phi^{(1)}(\beta_{mr}\eta)+\frac{\mu+\lambda}{2\mu+\lambda}\Psi^{(1)}(\beta_{mr}\eta)\Big\}\\
&+G'_{mr}\Big\{\frac{\mu}{2\mu+\lambda}\phi^{(2)}(\beta_{mr}\eta)+\frac{\mu+\lambda}{2\mu+\lambda}\Psi^{(2)}(\beta_{mr}\eta)\Big\}\Big]\sin\frac{m\pi}{a}x\cos\frac{r\pi}{c}z\\
&+\sum_n\sum_r\frac{a}{2}\Big[J_{nr}\Big\{Q^{(1)}(\alpha_{nr}\xi)-\frac{\mu+\lambda}{2\mu+\lambda}\frac{n^2a^2}{b^2\alpha_2^{nr}}P^{(1)}(\alpha_{nr}\xi)\Big\}+J'_{nr}\Big\{Q^{(2)}(\alpha_{nr}\xi)\\
&-\frac{\mu+\lambda}{2\mu+\lambda}\frac{n^2a^2}{b^2\alpha_2^{nr}}P^{(2)}(\alpha_{nr}\xi)\Big\}\Big]\cos\frac{m\pi}{a}x\cdot\cos\frac{r\pi}{c}z-\sum_m\sum_n\frac{b^2m}{4a\beta_{mr}}\\
&\times\Big[H_{mr}\Big\{\frac{\mu}{2\mu+\lambda}\phi^{(1)}(\beta_{mr}\eta)+\frac{\mu+\lambda}{2\mu+\lambda}\Psi^{(1)}(\beta_{mr}\eta)\Big\}+H'_{mr}\Big\{\frac{\mu}{2\mu+\lambda}\phi^{(2)}(\beta_{mr}\eta)\\
&+\frac{\mu+\lambda}{2\mu+\lambda}\Psi^{(2)}(\beta_{mr}\eta)\Big\}\Big]\sin\frac{n\pi}{b}y\cdot\cos\frac{r\pi}{c}z-\sum_m\sum_n\frac{c^4mn}{4ab(\gamma_{mn}^3)}\\
&\times\frac{\mu+\lambda}{2\mu+\lambda}\Big[D_{mn}\Big\{\phi^{(1)}(\gamma_{mn}\zeta)+\Psi^{(1)}(\gamma_{mn}\zeta)\Big\}+D'_{mn}\Big\{\phi^{(1)}(\gamma_{mn}\zeta)+\Psi^{(1)}(\gamma_{mn}\zeta)\Big\}\Big]
\end{aligned}
$$

$$
\left.
\begin{aligned}
&\times \cos\frac{m\pi}{a}x\cdot\sin\frac{n\pi}{b}y+\sum_m\sum_r\frac{b^2}{4\beta_{mr}}\Bigg[B_{mr}\left\{\frac{3\mu+\lambda}{\mu(2\mu+\lambda)}\phi^{(1)}(\beta_{mr}\eta)\right.\\
&\left.+\frac{\mu+\lambda}{\mu(2\mu+\lambda)}\Psi^{(1)}(\beta_{mr}\eta)\right\}+B'_{mr}\left\{\frac{3\mu+\lambda}{\mu(2\mu+\lambda)}\phi^{(2)}(\beta_{mr}\eta)+\frac{\mu+\lambda}{\mu(2\mu+\lambda)}\right.\\
&\left.\times\Psi^{(2)}(\beta_{mr}\eta)\right\}\Bigg]\sin\frac{m\pi}{a}x\cdot\cos\frac{r\pi}{c}z+\sum_m\sum_n\frac{c^2}{4\gamma_{mn}\mu}\Bigg[F_{mn}\left\{\left(2-\frac{\mu+\lambda}{2\mu+\lambda}\right.\right.\\
&\left.\left.\times\frac{c^2n^2}{b^2\gamma^2_{mn}}\right)\phi^{(1)}(\gamma_{mn}\zeta)-\frac{\mu+\lambda}{2\mu+\lambda}\frac{c^2n_2}{b^2\gamma_2^{mn}}\Psi^{(1)}(\gamma_{mn}\zeta)\right\}+F'_{mn}\left\{\left(2-\frac{\mu+\lambda}{2\mu+\lambda}\right.\right.\\
&\left.\left.\times\frac{c^2n^2}{b^2\gamma^2_{mn}}\right)\phi^{(2)}(\gamma_{mn}\zeta)-\frac{\mu+\lambda}{2\mu+\lambda}\frac{c^2n^2}{b^2\gamma_2^{mn}}\Psi^{(2)}(\gamma_{mn}\zeta)\right\}\Bigg]\sin\frac{m\pi}{a}x\cos\frac{n\pi}{b}y\\
&-\frac{b^2}{8\mu}\sum_m\frac{a}{bm}\Bigg[B_{m0}\left\{\frac{3\mu+\lambda}{2\mu+\lambda}\phi^{(1)}(\beta_{m0}\eta)+\frac{\mu+\lambda}{2\mu+\lambda}\Psi^{(1)}\beta_{m0}\eta)\right\}+B'_{m0}\\
&\times\left\{\frac{3\mu+\lambda}{2\mu+\lambda}\phi^{(2)}(\beta_{m0}\eta)+\frac{\mu+\lambda}{2\mu+\lambda}\Psi^{(2)}(\beta_{m0}\eta)\right\}\Bigg]\sin\frac{m\pi}{a}x+\frac{a}{4}\sum_n\Bigg[J_{n0}\\
&\times\left\{Q^{(1)}(\alpha_{n0}\xi)-\frac{\mu+\lambda}{2\mu+\lambda}P^{(1)}(\alpha_{n0}\xi)\right\}+J'_{n0}\left\{Q^{(2)}(\alpha_{n0}\xi)-\frac{\mu+\lambda}{2\mu+\lambda}P^{(2)}(\alpha_{n0}\xi)\right\}\Bigg]\\
&\times\cos\frac{n\pi}{b}y+\frac{b}{4}\sum_m\Bigg[H_{m0}\left\{\frac{\mu}{2\mu+\lambda}\phi^{(1)}(\beta_{m0}\eta)+\frac{\mu+\lambda}{2\mu+\lambda}\Psi^{(1)}(\beta_{m0}\eta)\right\}\\
&+H'_{m0}\left\{\frac{\mu}{2\mu+\lambda}\phi^{(2)}(\beta_{m0}\eta)+\frac{\mu+\lambda}{2\mu+\lambda}\Psi^{(2)}(\beta_{m0}\eta)\right\}\Bigg]\sin\frac{m\pi}{a}x-\sum_m\frac{ba}{8n}\\
&\times\frac{\mu+\lambda}{\mu(2\mu+\lambda)}\left\{A_{n0}\times P^{(1)}(\alpha_{n0}\xi)+A'_{n0}\times P^{(2)}(\alpha_{n0}\xi)\right\}\cos\frac{n\pi}{b}y+\sum_m\frac{ac}{4m}\\
&\times\left\{F_{m0}\phi^{(1)}(\gamma_{m0}\zeta)+F'_{m0}\phi^{(2)}(\gamma_{m0}\zeta)\right\}\sin\frac{m\pi}{a}x-\frac{a}{4}\sum_r\left\{H_{0r}Q^{(1)}(\beta_{0r}\eta)\right.\\
&\left.+H'_{0r}Q^{(2)}(\beta_{0r}\eta)\right\}\cos\frac{r\pi}{c}z-\frac{a}{8}\left\{H_{00}(1-2\xi)+H_{00}\right\}\\
&\qquad m,\ n,\ r=1,\ 2,\ 3,\ 4,\cdots\cdots,
\end{aligned}
\right\}\quad(65)
$$

$$
\begin{aligned}
w=&-\sum_m\sum_n\frac{c}{2}\Bigg[K_{mn}\left\{Q^{(1)}(\gamma_{mn}\zeta)+\frac{\mu+\lambda}{2\mu+\lambda}P^{(1)}(\gamma_{mn}\zeta)\right\}+K'_{mn}\left\{Q^{(2)}(\gamma_{mn}\zeta)\right.\\
&\left.+\frac{\mu+\lambda}{2\mu+\lambda}P^{(2)}(\gamma_{mn}\zeta)\right\}\Bigg]\sin\frac{m\pi}{a}x\cdot\sin\frac{n\pi}{b}y+\sum_n\sum_r\frac{a}{2}\Bigg[E_{nr}\left\{Q^{(1)}(\alpha_{nr}\xi)\right.\\
&\left.-\frac{\mu+\lambda}{2\mu+\lambda}\frac{r^2a^2}{c^2\alpha^2_{nr}}P^{(1)}(\alpha_{nr}\xi)\right\}+E'_{nr}\left\{Q^{(2)}(\alpha_{nr}\xi)-\frac{\mu+\lambda}{2\mu+\lambda}\frac{r^2a^2}{c^2\alpha^2_{nr}}P^{(2)}(\alpha_{nr}\xi)\right\}\Bigg]\\
&\times\sin\frac{n\pi}{b}y\cdot\sin\frac{r\pi}{c}z+\sum_m\sum_r\frac{b}{2}\Bigg[G_{mr}\left\{Q^{(1)}(\beta_{mr}\eta)-\frac{\mu+\lambda}{2\mu+\lambda}\frac{r^2b^2}{c^2\beta^2_{mr}}\right.\\
&\left.\times P^{(1)}(\beta_{mr}\eta)\right\}+G'_{mr}\left\{Q^{(2)}(\beta_{mr}\eta)-\frac{\mu+\lambda}{2\mu+\lambda}\frac{r^2b^2}{c^2\beta^2_{mr}}P^{(2)}(\beta_{mr}\eta)\right\}\Bigg]\sin\frac{m\pi}{a}\\
&\times x\cdot\sin\frac{r\pi}{c}z+\sum_n\sum_r\frac{a^3r}{4c\alpha^2_{nr}}\frac{\mu+\lambda}{\mu(2\mu+\lambda)}\left\{A_{nr}P^{(1)}(\alpha_{nr}\xi)+A'_{nr}P^{(2)}(\alpha_{nr}\xi)\right\}
\end{aligned}
$$

$$
\left.\begin{aligned}
&\times \sin\frac{n\pi}{b}y\sin\frac{r\pi}{c}z+\sum_m\sum_n\frac{c^3m}{4a\gamma_{mn}^2}\frac{\mu+\lambda}{2\mu+\lambda}\Big\{D_{mn}P^{(1)}(\gamma_{mn}\zeta)\\
&+D'_{mn}P^{(2)}(\gamma_{mn}\zeta)\Big\}\sin\frac{m\pi}{a}x\cdot\sin\frac{n\pi}{b}y-\sum_n\sum_r\frac{a^3nr}{2bc\alpha_{nr}^2}\frac{\mu+\lambda}{2\mu+\lambda}\\
&\times\Big\{J_{nr}P^{(1)}(\alpha_{nr}\xi)+J'_{nr}P^{(2)}(\alpha_{nr}\xi)\Big\}\sin\frac{n\pi}{b}y\cdot\sin\frac{r\pi}{c}z\\
&+\sum_m\sum_n\frac{b^3r}{4c\beta_{mr}^2}\frac{\mu+\lambda}{\mu(2\mu+\lambda)}\Big\{B_{mr}P^{(1)}(\beta_{mr}\eta)+B'_{mr}P^{(2)}(\beta_{mr}\eta)\Big\}\sin\frac{m\pi}{a}x\cdot\sin\\
&\times\frac{r\pi}{c}z+\sum_m\sum_n\frac{c^3n}{4b\gamma_{mn}^2}\frac{\eta+\lambda}{2\mu+\lambda}\Big\{F_{mn}P^{(1)}(\gamma_{mn}\zeta)+F'_{mn}P^{(2)}(\gamma_{mn}\zeta)\Big\}\sin\\
&\times\frac{m\pi}{a}x\cdot\sin\frac{n\pi}{b}y+\sum_m\sum_r\frac{b^3mr}{2ac\beta_{mr}^2}\frac{\mu+\lambda}{2\mu+\lambda}\Big\{H_{mr}P^{(1)}(\beta_{mr}\eta)\\
&+H'_{mr}P^{(2)}(\beta_{mr}\eta)\Big\}\sin\frac{m\pi}{a}x\cdot\sin\frac{r\pi}{c}z,\\
&\qquad m,\ n,\ r=1,\ 2,\ 3,\ 4,\cdots\cdots,
\end{aligned}\right\}\quad(66)
$$

The dilatation is, therefore, written in the form:

$$
\left.\begin{aligned}
&\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=e=\sum_m\sum_n\frac{\mu}{2\mu+\lambda}\gamma_{mn}\Big\{K_{mn}\phi^{(1)}(\gamma_{mn}\zeta)\\
&+K'_{mn}\phi^{(2)}(\gamma_{mn}\zeta)\Big\}\sin\frac{m\pi}{a}x\cdot\sin\frac{n\pi}{b}y+\sum_n\sum_r\frac{\mu}{2\mu+\lambda}\frac{ar}{c}\Big\{E_{nr}Q^{(1)}(\alpha_{nr}\xi)\\
&+E'_{nr}Q^{(2)}(\alpha_{nr}\xi)\Big\}\sin\frac{n\pi}{b}y\cdot\cos\frac{r\pi}{c}z+\sum_m\sum_r\frac{\mu}{2\mu+\lambda}\frac{br}{c}\Big\{G_{mr}Q^{(1)}(\beta_{mr}\eta)\\
&+G'_{mr}Q^{(2)}(\beta_{mr}\eta)\Big\}\sin\frac{m\pi}{a}x\cdot\cos\frac{r\pi}{c}z-\sum_m\sum_r\frac{bm}{a}\frac{\mu}{2\mu+\lambda}\Big\{H_{mr}Q^{(1)}(\beta_{mr}\eta)\\
&+H'_{mr}Q^{(2)}(\beta_{mr}\eta)\Big\}\sin\frac{m\pi}{a}x\cdot\cos\frac{r\pi}{c}z-\sum_n\sum_r\frac{an}{b}\frac{\mu}{2\mu+\lambda}\Big\{J_{nr}Q^{(1)}(\alpha_{nr}\xi)\\
&+J'_{nr}Q^{(2)}(\alpha_{nr}\xi)\Big\}\sin\frac{n\pi}{b}y\cdot\cos\frac{r\pi}{c}z-\sum_m\sum_r\frac{b}{2\mu+\lambda}\Big\{B_{mr}Q^{(1)}(\beta_{mr}\eta)\\
&+B'_{mr}Q^{(2)}(\beta_{mr}\eta)\Big\}\sin\frac{m\pi}{a}x\cdot\cos\frac{r\pi}{c}z-\sum_n\sum_r\frac{a}{2\mu+\lambda}\Big\{A_{nr}Q^{(1)}(\beta_{nr}\xi)\\
&+A'_{nr}Q^{(2)}(\alpha_{nr}\xi)\Big\}\sin\frac{n\pi}{b}y\cdot\cos\frac{r\pi}{c}z-\sum_n\sum_r\frac{c^2n}{2b\gamma_{mn}}\frac{1}{2\mu+\lambda}\\
&\times\Big\{F_{mn}\phi^{(1)}(\gamma_{mn}\zeta)+F'_{mn}\phi^{(2)}(\gamma_{mn}\zeta)\Big\}\sin\frac{m\pi}{a}x\cdot\sin\frac{n\pi}{a}y-\sum_m\sum_n\frac{c^2m}{2a\gamma_{mn}}\\
&\times\frac{1}{2\mu+\lambda}\Big\{D_{mn}\phi^{(1)}(\gamma_{mn}\zeta)+D'_{mn}\phi^{(2)}(\gamma_{mn}\zeta)\Big\}\sin\frac{m\pi}{a}x\cdot\cos\frac{r\pi}{c}z\\
&-\sum_n\frac{a}{4}\frac{1}{2\mu+\lambda}\Big\{A_{n0}Q^{(1)}(\alpha_{n0}\xi)+A'_{n0}Q^{(2)}(\alpha_{n0}\xi)\Big\}\sin\frac{n\pi}{b}y\\
&-\sum_m\frac{b}{4}\frac{1}{2\mu+\lambda}\Big\{B_{m0}Q^{(1)}(\beta_{m0}\eta)+B'_{m0}Q^{(2)}(\beta_{m0}\eta)\Big\}\sin\frac{m\pi}{a}x
\end{aligned}\right\}\quad(67)
$$

$$
\left.
\begin{aligned}
&-\sum_m \frac{bm}{2a}\frac{\mu}{2\mu+\lambda}\left\{H_{m0}Q^{(1)}(\beta_{m0}\eta)+H'_{m0}Q^{(2)}(\beta_{m0}\eta)\right\}\sin\frac{m\pi}{a}x\\
&-\sum_n \frac{an}{2b}\frac{\mu}{2\mu+\lambda}\left\{J_{n0}Q^{(1)}(\alpha_{n0}\xi)+J'_{n0}Q^{(2)}(\alpha_{n0}\xi)\right\}\sin\frac{n\pi}{b}y\,.
\end{aligned}
\right\}
$$

Then, the relations (4) and (5) yield the component of stresses as follows:

$$
\left.
\begin{aligned}
\sigma_x &= 2\mu\frac{\partial u}{\partial x}+\lambda e\\
&=\sum_m\sum_n\Bigg[K_{mn}\left\{\left(\frac{\mu^2}{2\mu+\lambda}\frac{c^2m^2}{a^2\gamma_{mn}^2}+\frac{\lambda\mu}{2\mu+\lambda}\gamma_{mn}\right)\phi^{(1)}(\gamma_{mn}\zeta)\right.\\
&\left.-\frac{\mu(\mu+\lambda)}{2\mu+\lambda}\frac{c^2m^2}{a^2\gamma_{mn}^2}\Psi^{(1)}(\gamma_{mn}\zeta)\right\}+K'_{mn}\left\{\left(\frac{\mu^2}{2\mu+\lambda}\frac{c^2m^2}{a^2\gamma_{mn}^2}+\frac{\lambda\mu}{2\mu+\lambda}\gamma_{mn}\right)\right.\\
&\left.\times\phi^{(2)}(\gamma_{mn}\zeta)-\frac{\mu(\mu+\lambda)}{2\mu+\lambda}\frac{c^2m^2}{a^2\gamma_{mn}^2}\Psi^{(2)}(\gamma_{mn}\zeta)\right\}\Bigg]\sin\frac{m\pi}{a}x\cdot\sin\frac{n\pi}{b}y\\
&+\sum_m\sum_r\frac{br}{c}\Bigg[G_{mr}\left\{\frac{\lambda\mu}{2\mu+\lambda}Q^{(1)}(\beta_{mr}\eta)-\frac{b^2m^2}{a^2\beta_2^{mr}}\frac{\mu(\lambda+\mu)}{2\mu+\lambda}P^{(1)}(\beta_{mr}\eta)\right\}\\
&+G'_{mr}\left\{\frac{\lambda\mu}{2\mu+\lambda}Q^{(2)}(\beta_{mr}\eta)-\frac{b^2m^2}{a^2\beta_{mr}^2}\frac{\mu(\lambda+\mu)}{2\mu+\lambda}P^{(2)}(\beta_{mr}\eta)\right\}\Bigg]\sin\frac{m\pi}{a}x\\
&\times\cos\frac{r\pi}{c}z+\sum_n\sum_r\frac{ra}{c}\frac{\mu(\mu+\lambda)}{2\mu+\lambda}\left\{E_{nr}P^{(1)}(\alpha_{nr}\xi)+E'_{nr}P^{(2)}(\alpha_{nr}\xi)\right\}\\
&\times\sin\frac{n\pi}{b}y\cos\frac{r\pi}{c}z-\sum_m\sum_r\frac{bm}{a}\Bigg[H_{mr}\left\{\frac{\mu(3\mu+\lambda)}{2\mu+\lambda}Q^{(1)}(\beta_{mr}\eta)\right.\\
&\left.+\frac{\lambda(\mu+\lambda)}{2\mu+\lambda}\frac{b^2m^2}{a^2\beta_{mr}^2}P^{(1)}(\beta_{mr}\eta)\right\}+H'_{mr}\left\{\frac{\mu(3\mu+\lambda)}{2\mu+\lambda}Q^{(2)}(\beta_{mr}\eta)\right.\\
&\left.+\frac{\lambda(\mu+\lambda)}{2\mu+\lambda}\frac{b^2m^2}{a^2\beta_{mr}^2}P^{(2)}(\beta_{mr}\eta)\right\}\Bigg]\sin\frac{m\pi}{a}x\cdot\cos\frac{r\pi}{c}z-\sum_m\sum_r\frac{an}{b}\\
&\times\frac{\mu(\mu+\lambda)}{2\mu+\lambda}\left\{J_{nr}P^{(1)}(\alpha_{nr}\xi)+J'_{nr}P^{(2)}(\alpha_{nr}\xi)\right\}\sin\frac{n\pi}{b}y\cdot\cos\frac{r\pi}{c}z-\sum_m\sum_r\\
&\times\frac{b}{2}\Bigg[B_{mr}\left\{\frac{\lambda}{2\mu+\lambda}Q^{(1)}(\beta_{mr}\eta)-\frac{\mu+\lambda}{2\mu+\lambda}\frac{b^2m^2}{a^2\beta_{mr}^2}P^{(1)}(\beta_{mr}\eta)\right\}+B'_{mr}\\
&\times\left\{\frac{\lambda}{2\mu+\lambda}Q^{(2)}(\beta_{mr}\eta)-\frac{\mu+\lambda}{2\mu+\lambda}\frac{b^2m^2}{a^2\beta_{mr}^2}P^{(2)}(\beta_{mr}\eta)\right\}\Bigg]\sin\frac{m\pi}{a}x\cdot\cos\frac{r\pi}{c}z\\
&-\sum_m\sum_n\frac{c^2n}{2b\gamma_{mn}}\Bigg[F_{mn}\left\{\left(\frac{\lambda}{2\mu+\lambda}+\frac{\lambda(\mu+\lambda)}{\mu(2\mu+\lambda)}\frac{c^2m^2}{a^2\gamma_{mn}^2}\right)\phi^{(1)}(\gamma_{mn}\zeta)\right.\\
&\left.-\frac{c^2m_2}{a^2\gamma_{mn}^2}\frac{\mu+\lambda}{2\mu+\lambda}\Psi^{(1)}(\gamma_{mn}\zeta)\right\}+F'_{mn}\left\{\left(\frac{\lambda}{2\mu+\lambda}+\frac{\lambda(\mu+\lambda)}{\mu(2\mu+\lambda)}\frac{c^2m^2}{a^2\gamma_{mn}^2}\right)\right.\\
&\left.\times\phi^{(2)}(\gamma_{mn}\zeta)-\frac{c^2m^2}{a^2\gamma_{mn}^2}\frac{\mu+\lambda}{2\mu+\lambda}\Psi^{(2)}(\gamma_{mn}\zeta)\right\}\Bigg]\sin\frac{m\pi}{a}x\cdot\sin\frac{n\pi}{b}y\\
&-\sum_n\sum_r\frac{a}{2}\Bigg[A_{nr}\left\{Q^{(1)}(\alpha_{nr}\xi)+\frac{\mu+\lambda}{2\mu+\lambda}P^{(1)}(\alpha_{nr}\xi)\right\}+A'_{nr}\left\{Q^{(2)}(\alpha_{nr}\xi)\right.
\end{aligned}
\right\}\quad(68)
$$

$$+\frac{\mu+\lambda}{2\mu+\lambda}P^{(2)}(\alpha_{nr}\xi)\Big\}\Big]\sin\frac{n\pi}{b}y\cdot\cos\frac{r\pi}{c}z-\sum_m\sum_n\frac{c^2m}{2a\gamma_{mn}}\Big[D_{mn}$$
$$\times\Big\{\Big(\frac{4\mu+3\lambda}{2\mu+\lambda}-\frac{\mu+\lambda}{2\mu+\lambda}\frac{c^2m^2}{a^2\gamma_{mn}^2}\Big)\times\phi^{(1)}(\gamma_{mn}\zeta)-\frac{\mu+\lambda}{2\mu+\lambda}\frac{c^2m^2}{a^2\gamma_{mn}^2}\Psi^{(1)}(\gamma_{mn}\zeta)\Big\}$$
$$-D'_{mn}\Big\{\Big(\frac{4\mu+3\lambda}{2\mu+\lambda}-\frac{\mu+\lambda}{2\mu+\lambda}\frac{c^2m^2}{a^2\gamma_{mn}^2}\Big)\phi^{(2)}(\gamma_{mn}\zeta)-\frac{\mu+\lambda}{2\mu+\lambda}\frac{c^2m^2}{a^2\gamma_{mn}^2}$$
$$\times\Psi^{(2)}(\gamma_{mn}\zeta)\Big\}\Big]\sin\frac{m\pi}{a}x\times\sin\frac{n\pi}{b}y-\sum_n\frac{a}{4}\Big[A_{n0}\Big\{Q^{(1)}(\alpha_{n0}\xi)$$
$$+\frac{\mu+\lambda}{2\mu+\lambda}P^{(1)}(\alpha_{n0}\xi)\Big\}+A'_{n0}\Big\{Q^{(2)}(\alpha_{n0}\xi)+\frac{\mu+\lambda}{2\mu+\lambda}P^{(2)}(\alpha_{n0}\xi)\Big\}\Big]\sin\frac{n\pi}{b}y$$
$$-\sum_m\frac{bm}{2a}\Big[H_{m0}\Big\{\frac{\mu(3\mu+\lambda)}{2\mu+\lambda}Q^{(1)}(\beta_{m0}\eta)+\frac{\lambda(\mu+\lambda)}{2\mu+\lambda}\times P^{(1)}(\beta_{m0}\eta)\Big\}$$
$$+H'_{m0}\Big\{\frac{\mu(3\mu+\lambda)}{2\mu+\lambda}Q^{(2)}(\beta_{m0}\eta)+\frac{\lambda(\mu+\lambda)}{2\mu+\lambda}P^{(2)}(\beta_{m0}\eta)\Big\}\Big]\sin\frac{m\pi}{a}x$$
$$-\sum_n\frac{an}{2b}\frac{\mu(\mu+\lambda)}{2\mu+\lambda}\Big\{J_{n0}P^{(1)}(\alpha_{n0}\xi)+J'_{n0}P^{(2)}(\alpha_{n0}\xi)\Big\}\sin\frac{n\pi}{b}y$$
$$-\sum_m\frac{b}{4}\Big[B_{m0}\Big\{\frac{\lambda}{2\mu+\lambda}Q^{(1)}(\beta_{m0}\eta)-\frac{\mu+\lambda}{2\mu+\lambda}P^{(1)}(\beta_{m0}\eta)\Big\}$$
$$+B'_{m0}\Big\{\frac{\lambda}{2\mu+\lambda}Q^{(2)}(\beta_{m0}\eta)-\frac{\mu+\lambda}{2\mu+\lambda}P^{(2)}(\beta_{m0}\eta)\Big\}\Big]\sin\frac{m\pi}{a}x\,,$$

$$\sigma_y=2\mu\frac{\partial v}{\partial y}+\lambda e=\sum_m\sum_n\Big[K_{mn}\Big\{\Big(\frac{\mu^2}{2\mu+\lambda}\frac{c^2n^2}{b^2\gamma_{mn}}+\frac{\mu\lambda}{2\mu+\lambda}\gamma_{mn}\Big)\phi^{(1)}(\gamma_{mn}\zeta)$$
$$-\frac{\mu(\mu+\lambda)}{2\mu+\lambda}\frac{c^2n^2}{b^2\gamma_{mn}}\Psi^{(1)}(\gamma_{mn}\zeta)\Big\}+K'_{mn}\Big\{\Big(\frac{\mu^2}{2\mu+\lambda}\frac{c^2n^2}{b^2\gamma_{mn}}+\frac{\mu\lambda}{2\mu+\lambda}\gamma_{mn}\Big)$$
$$\times\phi^{(2)}(\gamma_{mn}\zeta)-\frac{\mu(\mu+\lambda)}{2\mu+\lambda}\frac{c^2n^2}{b^2\gamma_{mn}}\Psi^{(2)}(\gamma_{mn}\zeta)\Big\}\Big]\sin\frac{m\pi}{a}x\cdot\sin\frac{n\pi}{b}y$$
$$-\sum_n\sum_r\frac{ar}{c}\Big[E_{nr}\Big\{\frac{a^2n^2}{b^2\alpha_{nr}^2}\frac{\mu(\mu+\lambda)}{2\mu+\lambda}P^{(1)}(\alpha_{nr}\xi)-\frac{\mu\lambda}{2\mu+\lambda}Q^{(1)}(\alpha_{nr}\xi)\Big\}$$
$$+E'_{nr}\Big\{\frac{a^2n^2}{b^2\alpha_{nr}^2}\frac{\mu(\mu+\lambda)}{2\mu+\lambda}P^{(2)}(\alpha_{nr}\xi)-\frac{\mu\lambda}{2\mu+\lambda}Q^{(2)}\alpha(_{nr}\xi)\Big\}\Big]\sin\frac{n\pi}{b}y$$
$$\times\cos\frac{r\pi}{c}z+\sum_m\sum_r\frac{rb}{c}\frac{\mu(\mu+\lambda)}{2\mu+\lambda}\times\Big\{G_{mr}P^{(1)}(\beta_{mr}\eta)+G'_{mr}P^{(2)}(\beta_{mr}\eta)\Big\}$$
$$\times\sin\frac{m\pi}{a}x\cdot\cos\frac{r\pi}{c}z+\sum_n\sum_r\frac{an}{b}\Big[J_{nr}\Big\{\frac{\mu(3\mu+\lambda)}{2\mu+\lambda}Q^{(1)}(\alpha_{nr}\xi)$$
$$+\frac{\lambda(\mu+\lambda)}{2\mu+\lambda}\frac{a^2n^2}{b^2\alpha_{nr}^2}P^{(1)}(\alpha_{nr}\xi)\Big\}+J'_{nr}\Big\{\frac{\mu(3\mu+\lambda)}{2\mu+\lambda}Q^{(2)}(\alpha_{nr}\xi)$$
$$+\frac{\lambda(\mu+\lambda)}{2\mu+\lambda}\frac{a^2n^2}{b^2\alpha_{nr}^2}P^{(2)}(\alpha_{nr}\xi)\Big\}\Big]\sin\frac{n\pi}{b}y\cdot\cos\frac{r\pi}{c}z-\sum_n\sum_r\frac{am}{b}\frac{\mu(\mu+\lambda)}{2\mu+\lambda}$$
$$\times\Big\{H_{mr}P^{(1)}(\beta_{mr}\eta)+H'_{mr}P^{(2)}(\beta_{mr}\eta)\Big\}\sin\frac{m\pi}{a}x\cdot\cos\frac{r\pi}{c}z-\sum_n\sum_r\frac{a}{2}$$

$$
\left.\begin{aligned}
&\times\left[A_{nr}\left\{\frac{\lambda}{2\mu+\lambda}Q^{(1)}(\alpha_{nr}\xi)-\frac{\mu+\lambda}{2\mu+\lambda}\frac{a^2n^2}{b^2\alpha_{nr}^2}P^{(1)}(\alpha_{nr}\xi)\right\}+A'_{nr}\right.\\
&\left.\times\left\{\frac{\lambda}{2\mu+\lambda}Q^{(2)}(\alpha_{nr}\xi)-\frac{\mu+\lambda}{2\mu+\lambda}\frac{a^2n^2}{b^2\alpha_{nr}^2}P^{(2)}(\alpha_{nr}\xi)\right\}\right]\sin\frac{n\pi}{b}y\cos\frac{r\pi}{c}z\\
&-\sum_m\sum_r\frac{c^2m}{2a\gamma_{mn}}\left[D_{mn}\left\{\left(\frac{\lambda}{2\mu+\lambda}+\frac{\lambda(\mu+\lambda)}{\mu(2\mu+\lambda)}\frac{c^2n^2}{b^2\gamma_{mn}^2}\right)\phi^{(1)}(\gamma_{mn}\zeta)-\frac{c^2n^2}{b^2\gamma_{mn}^2}\right.\right.\\
&\times\left.\frac{\mu+\lambda}{2\mu+\lambda}\Psi^{(1)}(\gamma_{mn}\zeta)\right\}+D'_{mn}\left\{\left(\frac{\lambda}{2\mu+\lambda}+\frac{\lambda(\mu+\lambda)}{\mu(2\mu+\lambda)}\frac{c^2n^2}{b^2\gamma_{mn}^2}\right)\phi^{(2)}(\gamma_{mn}\zeta)\right.\\
&\left.\left.-\frac{c^2n^2}{b^2\gamma_{mn}^2}\frac{\mu+\lambda}{2\mu+\lambda}\Psi^{(2)}(\gamma_{mn}\zeta)\right\}\right]\sin\frac{m\pi}{a}x\cdot\sin\frac{n\pi}{b}y-\sum_m\sum_r\frac{b}{2}\\
&\times\left[B_{mr}\left\{Q^{(1)}(\beta_{mr}\eta)+\frac{\mu+\lambda}{2\mu+\lambda}P^{(1)}(\beta_{mr}\eta)\right\}+B'_{mr}\left\{Q^{(2)}(\beta_{mr}\eta)\right.\right.\\
&\left.\left.+\frac{\mu+\lambda}{2\mu+\lambda}P^{(2)}(\beta_{mr}\eta)\right\}\right]\sin\frac{m\pi}{a}x\cdot\cos\frac{r\pi}{c}z-\sum_m\sum_n\frac{c^2m}{2b\gamma_{mn}}\left[F_{mn}\right.\\
&\times\left\{\left(\frac{4\mu+3\lambda}{2\mu+\lambda}-\frac{\mu+\lambda}{2\mu+\lambda}\frac{c^2n^2}{b^2\gamma_{mn}^2}\right)\phi^{(1)}(\gamma_{mn}\zeta)-\frac{\mu+\lambda}{2\mu+\lambda}\frac{c^2n^2}{b^2\gamma_{mn}^2}\Psi^{(1)}(\gamma_{mn}\zeta)\right\}\\
&\left.+F'_{mn}\left\{\left(\frac{4\mu+3\lambda}{2\mu+\lambda}-\frac{\mu+\lambda}{2\mu+\lambda}\frac{c^2n^2}{b^2\gamma_{mn}^2}\right)\phi^{(2)}(\gamma_{mn}\zeta)-\frac{\mu+\lambda}{2\mu+\lambda}\frac{c^2n^2}{b^2\gamma_{mn}^2}\Psi^{(1)}(\gamma_{mn}\zeta)\right\}\right]\\
&\times\sin\frac{m\pi}{a}x\sin\frac{n\pi}{b}y-\sum_m\frac{b}{4}\left[B_{m0}\left\{Q^{(1)}(\beta_{m0}\eta)+\frac{\mu+\lambda}{2\mu+\lambda}P^{(1)}(\beta_{m0}\eta)\right\}\right.\\
&\left.+B'_{m0}\left\{Q^{(2)}(\beta_{m0}\eta)+\frac{\mu+\lambda}{2\mu+\lambda}P^{(2)}(\beta_{m0}\eta)\right\}\right]\sin\frac{m\pi}{a}x-\sum_n\frac{an}{2b}\left[J_{n0}\right.\\
&\times\left\{\frac{\mu(3\mu+\lambda)}{2\mu+\lambda}Q^{(1)}(\alpha_{n0}\xi)+\frac{\lambda(\mu+\lambda)}{2\mu+\lambda}P^{(1)}(\alpha_{n0}\xi)\right\}+J'_{n0}\left\{\frac{\mu(3\mu+\lambda)}{2\mu+\lambda}Q^{(2)}(\alpha_{n0}\xi)\right.\\
&\left.\left.+\frac{\lambda(\mu+\lambda)}{2\mu+\lambda}P^{(2)}(\alpha_{n0}\xi)\right\}\right]\sin\frac{n\pi}{b}y-\sum_m\frac{bm}{2a}\frac{\mu(\mu+\lambda)}{2\mu+\lambda}\left\{H_{m0}P^{(1)}(\beta_{m0}\eta)\right.\\
&\left.+H'_{m0}P^{(2)}(\beta_{m0}\eta)\right\}\sin\frac{m\pi}{a}x-\sum_n\frac{a}{4}\left[A_{n0}\left\{\frac{\lambda}{2\mu+\lambda}Q^{(1)}(\alpha_{n0}\xi)-\frac{\mu+\lambda}{2\mu+\lambda}\right.\right.\\
&\left.\left.\times P^{(1)}(\alpha_{n0}\xi)\right\}+A'_{n0}\left\{\frac{\lambda}{2\mu+\lambda}Q^{(2)}(\alpha_{n0}\xi)-\frac{\mu+\lambda}{2\mu+\lambda}P^{(2)}(\alpha_{n0}\xi)\right\}\right]\sin\frac{n\pi}{b}y,
\end{aligned}\right\}\quad(69)
$$

$$
\begin{aligned}
\sigma_z&=2\mu\frac{\partial w}{\partial z}+\lambda e=\sum_m\sum_n\frac{\mu(\mu+\lambda)}{2\mu+\lambda}\gamma_{mn}\left[K_{mn}\left\{\phi^{(1)}(\gamma_{mn}\zeta)+\Psi^{(1)}(\gamma_{mn}\zeta)\right\}\right.\\
&\left.+K'_{mn}\left\{\phi^{(2)}(\gamma_{mn}\zeta)+\Psi^{(2)}(\gamma_{mn}\zeta)\right\}\right]\sin\frac{m\pi}{a}x\cdot\sin\frac{n\pi}{b}y+\sum_n\sum_r\frac{ar}{c}\\
&\times\left[E_{nr}\left\{\frac{2\mu(\mu+\lambda)}{2\mu+\lambda}Q^{(1)}(\alpha_{nr}\xi)-\frac{\mu(\mu+\lambda)}{2\mu+\lambda}\frac{r^2a^2}{c^2\alpha_{nr}^2}P^{(1)}(\alpha_{nr}\xi)\right\}+\left[E'_{nr}\right.\right.\\
&\left.\times\left\{\frac{2\mu(\mu+\lambda)}{2\mu+\lambda}Q^{(2)}(\alpha_{nr}\xi)-\frac{\mu(\mu+\lambda)}{2\mu+\lambda}\frac{r^2a^2}{c^2\alpha_{nr}^2}P^{(2)}(\alpha_{nr}\xi)\right\}\right]\sin\frac{n\pi}{b}y\cdot\cos\frac{r\pi}{c}z\\
&+\sum_m\sum_r\frac{br}{c}\left[G_{mr}\left\{\frac{2\mu(\mu+\lambda)}{2\mu+\lambda}Q^{(1)}(\beta_{mr}\eta)-\frac{\mu(\mu+\lambda)}{2\mu+\lambda}\right.\right.
\end{aligned}
$$

$$
\left.\begin{aligned}
&\times \frac{r^2b^2}{c^2\beta_{mr}^2} P^{(1)}(\beta_{mr}\mu)\Big\} + G'_{mr}\Big\{\frac{2\mu(\mu+\lambda)}{2\mu+\lambda} Q^{(2)}(\beta_{mr}\eta) - \frac{\mu(\mu+\lambda)}{2\mu+\lambda}\frac{r^2b^2}{c^2\beta_{mr}^2} \\
&\times P^{(1)}(\beta_{mr}\eta)\Big\}\Big]\sin\frac{m\pi}{a}x\cdot\cos\frac{r\pi}{c}z - \sum_n\sum_r \frac{a}{2}\Big[A_{nr}\Big\{\frac{\lambda}{2\mu+\lambda} Q^{(1)}(\alpha_{nr}\xi) \\
&- \frac{a^2r^2}{c^2}\frac{\mu+\lambda}{2\mu+\lambda} P^{(1)}(\alpha_{nr}\xi)\Big\} + A'_{nr}\Big\{\frac{\lambda}{2\mu+\lambda} Q^{(2)}(\alpha_{nr}\xi) - \frac{a^2r^2}{c^2}\frac{\mu+\lambda}{2\mu+\lambda} \\
&\times P^{(2)}(\alpha_{nr}\xi)\Big\}\Big]\sin\frac{n\pi}{b}y\cdot\cos\frac{r\pi}{c}z + \sum_m\sum_n \frac{c^2m}{4a\gamma_{mn}}\Big[D_{mn}\Big\{\frac{\mu}{2\mu+\lambda}\phi^{(1)}(\gamma_{mn}\zeta) \\
&- \frac{\mu+\lambda}{2\mu+\lambda}\Psi^{(1)}(\gamma_{mn}\zeta)\Big\} + D'_{mn}\Big\{\frac{\mu}{2\mu+\lambda}\phi^{(2)}(\gamma_{mn}\zeta) - \frac{\mu+\lambda}{2\mu+\lambda}\Psi^{(2)}(\gamma_{mn}\zeta)\Big\}\Big] \\
&\times \sin\frac{m\pi}{a}x\cdot\sin\frac{n\pi}{b}y - \sum_m\sum_n \frac{an}{b}\Big[J_{nr}\Big\{\frac{\mu\lambda}{2\mu+\lambda} Q^{(1)}(\alpha_{nr}\xi) + \frac{r^2a^2}{c^2} \\
&\times \frac{\mu(\mu+\lambda)}{2\mu+\lambda} P^{(1)}(\alpha_{nr}\xi)\Big\} + J'_{nr}\Big\{\frac{\mu\lambda}{2\mu+\lambda} Q^{(2)}(\alpha_{nr}\xi) + \frac{r^2a^2}{c^2}\frac{\mu(\mu+\lambda)}{2\mu+\lambda} \\
&\times P^{(2)}(\alpha_{nr}\xi)\Big\}\Big]\sin\frac{n\pi}{b}y\cdot\cos\frac{r\pi}{c}z - \sum_m\sum_r \frac{b}{2}\Big[B_{mr}\Big\{\frac{\lambda}{2\mu+\lambda} Q^{(1)}(\beta_{mr}\eta) \\
&+ \frac{b^2r^2}{c^2}\frac{\mu+\lambda}{2\mu+\lambda} P^{(1)}(\beta_{mr}\eta)\Big\} + B'_{mr}\Big\{\frac{\lambda}{2\mu+\lambda} Q^{(2)}(\beta_{mr}\eta) + \frac{b^2r^2}{c^2}\frac{\mu+\lambda}{2\mu+\lambda} \\
&\times P^{(2)}(\beta_{mr}\eta)\Big\}\Big]\sin\frac{m\pi}{a}x\cdot\cos\frac{r\pi}{c}z + \sum_m\sum_n \frac{c^2n}{4b\gamma_{mn}}\Big[F_{mn}\Big\{\frac{\mu}{2\mu+\lambda} \\
&\times \phi^{(1)}(\gamma_{mn}\zeta) - \frac{\mu+\lambda}{2\mu+\lambda}\Psi^{(1)}(\gamma_{mn}\zeta)\Big\} - F'_{mn}\Big\{\frac{\mu}{2\mu+\lambda}\phi^{(2)}(\gamma_{mn}\zeta) - \frac{\mu+\lambda}{2\mu+\lambda} \\
&\times \Psi^{(2)}(\gamma_{mn}\zeta)\Big\}\Big]\sin\frac{m\pi}{a}x\cdot\sin\frac{n\pi}{b}y - \sum_m\sum_r \frac{bm}{a}\Big[H_{mr}\Big\{\frac{\mu\lambda}{2\mu+\lambda} \\
&\times Q^{(1)}(\beta_{mr}\eta) + \frac{r^2b^2}{c^2\beta_{mr}^2}\frac{\mu(\mu+\lambda)}{2\mu+\lambda} P^{(1)}(\beta_{mr}\eta)\Big\} + H'_{mr}\Big\{\frac{\mu\lambda}{2\mu+\lambda} Q^{(2)}(\beta_{mr}\eta) \\
&+ \frac{b^2r^2}{c^2\beta_{mr}^2}\frac{\mu(\mu+\lambda)}{2\mu+\lambda} P^{(2)}(\beta_{mr}\eta)\Big\}\Big]\sin\frac{m\pi}{a}x\cdot\cos\frac{r\pi}{c}z - \sum_n \frac{a}{4}\frac{\lambda}{2\mu+\lambda} \\
&\times \Big\{A_{n0}Q^{(1)}(\alpha_{n0}\xi) + A'_{n0}Q^{(2)}(\alpha_{n0}\xi)\Big\}\sin\frac{n\pi}{b}y - \sum_n \frac{an}{2b}\frac{\lambda}{2\mu+\lambda}\Big\{J_{n0}Q(\alpha_{n0}\xi) \\
&+ J'_{n0}Q^{(2)}(\alpha_{n0}\xi)\Big\}\sin\frac{n\pi}{b}y - \sum_m \frac{bm}{2a}\frac{\mu\lambda}{2\mu+\lambda}\Big\{H_{m0}Q^{(1)}(\beta_{m0}\eta) \\
&+ H'_{m0}Q^{(2)}(\beta_{m0}\eta)\Big\}\sin\frac{m\pi}{a}x - \sum_m \frac{b}{4}\frac{\lambda}{2\mu+\lambda}\Big\{B_{m0}Q^{(1)}(\beta_{m0}\eta) \\
&+ B'_{m0}Q^{(2)}(\beta_{m0}\eta)\Big\}\sin\frac{m\pi}{a}x,
\end{aligned}\right\} \tag{70}
$$

$$
\begin{aligned}
\tau_{yz} = \mu\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) = &-\sum_m\sum_n \frac{cn}{b}\frac{\mu(\mu+\lambda)}{2\mu+\lambda}\Big\{K_{mn}P^{(1)}(\gamma_{mn}\zeta) \\
&+ K'_{mn}P^{(2)}(\gamma_{mn}\zeta)\Big\}\sin\frac{m\pi}{a}x\sin\frac{n\pi}{b}y - \sum_m\sum_r \frac{\pi\beta_{mr}}{2}\Big[G_{mr}\Big\{\phi^{(1)}(\beta_{mr}\eta)
\end{aligned}
$$

$$\left.\begin{aligned}
&-\frac{2(\mu+\lambda)}{2\mu+\lambda}\frac{b^2r^2}{c^2\beta_{mr}^2}\Psi^{(1)}(\beta_{mr}\eta)\Big\}+G'_{mr}\Big\{\phi^{(2)}(\beta_{mr}\eta)-\frac{2(\mu+\lambda)}{2\mu+\lambda}\frac{b^2r^2}{c^2\beta_{mr}^2}\\
&\times\Psi^{(2)}(\beta_{mr}\eta)\Big\}\Big]\sin\frac{m\pi}{a}x\cdot\sin\frac{r\pi}{c}z+\sum_n\sum_r\mu\frac{an}{2b}\Big[E_{nr}\Big\{Q^{(1)}(\alpha_{nr}\xi)\\
&-\frac{2(\mu+\lambda)}{2\mu+\lambda}\frac{a^2r^2}{c^2\alpha_{nr}^2}P^{(1)}(\alpha_{nr}\xi)\Big\}+E'_{nr}\Big\{Q^{(2)}(\alpha_{nr}\xi)-\frac{2(\mu+\lambda)}{2\mu+\lambda}\frac{a^2r^2}{c^2\alpha_{nr}^2}\\
&\times P^{(2)}(\alpha_{nr}\xi)\Big\}\Big]\cos\frac{n\pi}{b}y\sin\frac{r\pi}{b}z-\sum_n\sum_r\mu\frac{br}{2c}\Big[J_{nr}\Big\{Q^{(1)}(\alpha_{nr}\xi)\\
&-\frac{2(\mu+\lambda)}{2\mu+\lambda}\frac{n^2a^2}{b^2\alpha_{nr}^2}P^{(1)}(\alpha_{nr}\xi)\Big\}+J'_{nr}\Big\{Q^{(2)}(\alpha_{nr}\xi)-\frac{2(\mu+\lambda)}{2\mu+\lambda}\frac{n^2a^2}{b^2\alpha_{nr}^2}\\
&\times P^{(2)}(\alpha_{nr}\xi)\Big\}\Big]\cos\frac{n\pi}{b}y\cdot\sin\frac{r\pi}{c}z+\sum_m\sum_r\mu\frac{b^2mr}{2ac\beta_{mr}}\Big[H_{mr}\Big\{\frac{\mu}{2\mu+\lambda}\\
&\times\phi^{(1)}(\beta_{mr}\eta)+\frac{2(\mu+\lambda)}{2\mu+\lambda}\Psi^{(2)}(\beta_{mr}\eta)\Big\}+H'_{mr}\Big\{\frac{\mu}{2\mu+\lambda}\phi^{(2)}(\beta_{mr}\eta)\\
&+\frac{2(\mu+\lambda)}{2\mu+\lambda}\Psi^{(2)}(\beta_{mr}\eta)\Big\}\Big]\sin\frac{m\pi}{a}x\cdot\sin\frac{r\pi}{c}z+\sum_n\sum_r\mu\frac{a^3rn}{2bc}\frac{1}{\beta_{mr}^2}\\
&\times\frac{\mu+\lambda}{2\mu+\lambda}\Big\{A_{nr}P^{(1)}(\alpha_{nr}\xi)+A'_{nr}P^{(2)}(\alpha_{nr}\xi)\Big\}\cos\frac{n\pi}{b}y\cdot\sin\frac{r\pi}{c}z+\sum_m\sum_n\\
&\times\frac{c^3mn}{2ab\gamma_{mn}^2}\frac{\mu+\lambda}{2\mu+\lambda}\Big\{D_{mn}P^{(1)}(\gamma_{mn}\zeta)+D'_{mn}P^{(2)}(\gamma_{mn}\zeta)\Big\}\sin\frac{m\pi}{a}x\cdot\cos\\
&\times\frac{n\pi}{b}y-\sum_m\sum_r\frac{b^3r}{4c\beta_{mr}}\Big[B_{mr}\Big\{\frac{2\mu}{2\mu+\lambda}\phi^{(1)}(\beta_{mr}\eta)+\frac{2(\mu+\lambda)}{2\mu+\lambda}\Psi^{(1)}(\beta_{mr}\eta)\Big\}\\
&+B'_{mr}\Big\{\frac{2\mu}{2\mu+\lambda}\phi^{(2)}(\beta_{mr}\eta)+\frac{2(\mu+\lambda)}{2\mu+\lambda}\Psi^{(2)}(\beta_{mr}\eta)\Big\}\Big]\sin\frac{m\pi}{a}x\cdot\sin\\
&\times\frac{r\pi}{c}z-\sum_m\sum_n\frac{c}{2}\Big[F_{mn}\Big\{Q^{(1)}(\gamma_{mn}\zeta)-\frac{\mu+\lambda}{2\mu+\lambda}\frac{c^2n^2}{b^2\gamma_{mn}^2}P^{(1)}(\gamma_{mn}\zeta)\Big\}\\
&+F'_{mn}\Big\{Q^{(2)}(\gamma_{mn}\zeta)-\frac{\mu+\lambda}{2\mu+\lambda}\frac{c^2n^2}{b^2\gamma_{mn}^2}\times P^{(2)}(\gamma_{mn}\zeta)\Big\}\Big]\cos\frac{n\pi}{b}y\cdot\sin\frac{m\pi}{a}x\\
&-\sum_n\frac{c}{4}\Big\{F_{m0}Q^{(1)}(\gamma_{m0}\zeta)+F'_{m0}Q^{(2)}(\gamma_{m0}\zeta)\Big\}\sin\frac{n\pi}{b}y+\sum_r\frac{ar}{4c}\\
&\times\Big\{H_{0r}Q^{(1)}(\alpha_{0r}\xi)+H'_{0r}Q^{(2)}(\alpha_{0r}\xi)\Big\}\sin\frac{r\pi}{c}z,
\end{aligned}\right\}\quad(71)$$

$$\begin{aligned}
\tau_{zx}&=\mu\Big(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\Big)=-\sum_m\sum_n\frac{cm}{a}\frac{\mu(\mu+\lambda)}{2\mu+\lambda}\Big\{K_{mn}P^{(1)}(\gamma_{mn}\zeta)+K'_{mn}\\
&\times P^{(2)}(\gamma_{mn}\zeta)\Big\}\cdot\cos\frac{m\pi}{a}x\cdot\sin\frac{n\pi}{b}y-\sum_n\sum_r\mu\frac{\alpha_{nr}}{2}\Big[E_{nr}\Big\{\phi^{(1)}(\alpha_{nr}\xi)\\
&-\frac{2(\mu+\lambda)}{2\mu+\lambda}\frac{a^2r^2}{c^2\alpha_{nr}^2}\Psi^{(1)}(\alpha_{nr}\xi)\Big\}+E'_{nr}\Big\{\phi^{(2)}(\alpha_{nr}\xi)-\frac{2(\mu+\lambda)}{2\mu+\lambda}\frac{a^2r^2}{c^2\alpha_{nr}^2}\\
&\times\Psi^{(2)}(\alpha_{nr}\xi)\Big\}\Big]\sin\frac{n\pi}{b}y\cdot\sin\frac{r\pi}{c}z+\sum_r\sum_m\mu\frac{bm}{2a}\Big[G_{mr}\Big\{Q^{(1)}(\beta_{mn}\eta)
\end{aligned}$$

$$
\left.\begin{aligned}
&-\frac{2(\mu+\lambda)}{2\mu+\lambda}\frac{a^2r^2}{c^2\beta_{mr}^2}P^{(1)}(\beta_{mr}\eta)\Big\}+G'_{mr}\Big\{Q^{(2)}(\beta_{mr}\eta)-\frac{2(\mu+\lambda)}{2\mu+\lambda}\frac{a^2r^2}{c^2\beta_{mr}^2}\\
&\times P^{(2)}(\beta_{mr}\eta)\Big\}\Big]\cos\frac{m\pi}{a}x\cdot\sin\frac{r\pi}{c}z+\sum_n\sum_r\mu\frac{a^2nr}{2b\alpha_{nr}}\Big[J_{nr}\Big\{\frac{\mu}{2\mu+\lambda}\\
&\times\phi^{(1)}(\alpha_{nr}\xi)+\frac{2(\mu+\lambda)}{2\mu+\lambda}\Psi^{(1)}(\alpha_{nr}\xi)\Big\}+J'_{nr}\Big\{\frac{\mu}{2\mu+\lambda}\phi^{(2)}(\alpha_{nr}\xi)\\
&+\frac{2(\mu+\lambda)}{2\mu+\lambda}\Psi^{(2)}(\alpha_{nr}\xi)\Big\}\Big]\sin\frac{n\pi}{b}y\cdot\sin\frac{r\pi}{c}z+\sum_m\sum_n\mu\frac{ar}{2c}\Big[H_{mr}\\
&\times\Big\{Q^{(1)}(\beta_{mr}\eta)-\frac{2(\mu+\lambda)}{2\mu+\lambda}\frac{m^2b^2}{a^2\beta_{mr}^2}P^{(1)}(\beta_{mr}\eta)\Big\}+H'_{mr}\Big\{Q^{(2)}(\beta_{mr}\eta)\\
&-\frac{2(\mu+\lambda)}{2\mu+\lambda}\frac{m^2b^2}{a^2\beta_{mr}^2}P^{(2)}(\beta_{mr}\eta)\Big\}\Big]\cos\frac{m\pi}{a}x\cdot\sin\frac{r\pi}{c}z+\sum_m\sum_r\frac{b^3rm}{2ac\beta_{mr}^2}\\
&\times\frac{\mu+\lambda}{2\mu+\lambda}\Big\{B_{mr}P^{(1)}(\beta_{mr}\eta)+B'_{mr}P^{(2)}(\beta_{mr}\eta)\Big\}\cos\frac{m\pi}{a}x\cdot\sin\frac{r\pi}{c}z\\
&+\sum_m\sum_n\frac{c^3mn}{2ab\gamma_{mn}^2}\frac{\mu+\lambda}{2\mu+\lambda}\Big\{F_{mn}P^{(1)}(\gamma_{mn}\zeta)+F'_{mn}P^{(2)}(\gamma_{mn}\zeta)\Big\}\\
&\times\cos\frac{m\pi}{a}x\cdot\sin\frac{n\pi}{b}y-\sum_n\sum_r\frac{a^2r}{4c\alpha_{nr}}\Big[A_{nr}\Big\{\frac{2\mu}{2\mu+\lambda}\phi^{(1)}(\alpha_{nr}\xi)\\
&+\frac{2(\mu+\lambda)}{2\mu+\lambda}\Psi^{(1)}(\alpha_{nr}\xi)\Big\}+A'_{nr}\Big\{\frac{2\mu}{2\mu+\lambda}\phi^{(2)}(\alpha_{nr}\xi)+\frac{2(\mu+\lambda)}{2\mu+\lambda}\Psi^{(2)}(\alpha_{nr}\xi)\Big\}\Big]\\
&\times\sin\frac{n\pi}{b}y\cdot\sin\frac{r\pi}{c}z-\sum_m\sum_n\frac{c}{2}\Big[D_{mn}\Big\{Q^{(1)}(\gamma_{mn}\zeta)+\frac{\mu+\lambda}{2\mu+\lambda}P^{(1)}(\gamma_{mn}\zeta)\Big\}\\
&+D'_{mn}\Big\{Q^{(2)}(\gamma_{mn}\zeta)+\frac{\mu+\lambda}{2\mu+\lambda}P^{(2)}(\gamma_{mn}\zeta)\Big\}\Big]\cos\frac{m\pi}{a}x\cdot\sin\frac{n\pi}{b}y\\
&-\sum_n\frac{c}{4}\Big\{D_{0n}Q^{(1)}(\gamma_{0n}\zeta)+D'_{0n}Q^{(2)}(\gamma_{0n}\zeta)\Big\}\sin\frac{n\pi}{b}y+\sum_r\frac{br}{4c}\Big\{J_{0r}Q^{(1)}(\beta_{0r}\eta)\\
&+J'_{0r}Q^{(2)}(\beta_{0r}\eta)\Big\}\sin\frac{r\pi}{c}z,
\end{aligned}\right\}\quad(72)
$$

$$
\begin{aligned}
\tau_{xy}=\mu\Big(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\Big)&=-\sum_m\sum_n\mu\frac{c^2}{ab}\frac{mn}{\gamma_{mn}}\Big[K_{mn}\Big\{\frac{\mu}{2\mu+\lambda}\phi^{(1)}(\gamma_{mn}\zeta)\\
&+\frac{\mu+\lambda}{2\mu+\lambda}\Psi^{(1)}(\gamma_{mn}\zeta)\Big\}+K'_{mn}\Big\{\frac{\mu}{2\mu+\lambda}\phi^{(2)}(\gamma_{mn}\zeta)+\frac{\mu+\lambda}{2\mu+\lambda}\Psi^{(2)}(\gamma_{mn}\zeta)\Big\}\Big]\\
&\times\cos\frac{m\pi}{a}x\times\cos\frac{n\pi}{b}y+\sum_m\sum_r\mu\frac{b^3mr}{2ca\beta_{mr}}\Big[G_{mr}\Big\{\frac{\lambda}{2\mu+\lambda}\phi^{(1)}(\beta_{mr}\eta)\\
&-\frac{2(\mu+\lambda)}{2\mu+\lambda}\Psi^{(1)}(\beta_{mr}\eta)\Big\}+G'_{mr}\Big\{\frac{\lambda}{2\mu+\lambda}\phi^{(2)}(\beta_{mr}\eta)-\frac{2(\mu+\lambda)}{2\mu+\lambda}\Psi^{(2)}(\beta_{mr}\eta)\Big\}\Big]\\
&\times\cos\frac{m\pi}{a}x\cdot\cos\frac{r\pi}{c}z+\sum_n\sum_r\mu\frac{a^3nr}{2cb\alpha_{nr}}\Big[E_{nr}\Big\{\frac{\lambda}{2\mu+\lambda}\phi^{(1)}(\alpha_{nr}\xi)\\
&-\frac{2(\mu+\lambda)}{2\mu+\lambda}\Psi^{(1)}(\alpha_{nr}\xi)\Big)\Big\}+E'_{nr}\Big\{\frac{\lambda}{2\mu+\lambda}\phi^{(2)}(\alpha_{nr}\xi)-\frac{2(\mu+\lambda)}{2\mu+\lambda}
\end{aligned}
$$

$$
\left.\begin{aligned}
&\times \Psi^{(2)}(\alpha_{nr}\xi)\Big\}\Big]\cos\frac{n\pi}{b}y\cdot\cos\frac{r\pi}{c}z+\sum_m\sum_n\frac{c^2m}{2a\gamma_{mn}}\Big[F_{mn}\Big\{\Big(1-\frac{\mu+\lambda}{2\mu+\lambda}\\
&\times\frac{c^2n^2}{b^2\gamma^2_{mn}}\Big)\times\phi^{(1)}(\gamma_{mn}\zeta)-\frac{\mu+\lambda}{2\mu+\lambda}\frac{c^2n^2}{b^2\gamma^2_{mn}}\Psi^{(1)}(\gamma_{mn}\zeta)\Big\}+F'_{mn}\\
&\times\Big\{\Big(1-\frac{\mu+\lambda}{2\mu+\lambda}\frac{c^2n^2}{b^2\gamma^2_{mn}}\Big)\phi^{(2)}(\gamma_{mn}\zeta)-\frac{\mu+\lambda}{2\mu+\lambda}\frac{c^2n^2}{b^2\gamma^2_{mn}}\Psi^{(2)}(\gamma_{mn}\zeta)\Big\}\Big]\\
&\times\cos\frac{m\pi}{a}x\cdot\cos\frac{n\pi}{b}y+\sum_m\sum_n\frac{c^2n}{2b\gamma_{mn}}\Big[D_{mn}\Big\{\Big(1-\frac{\mu+\lambda}{2\mu+\lambda}\frac{c^2m^2}{a^2\gamma^2_{mn}}\Big)\\
&\times\phi^{(1)}(\gamma_{mn}\zeta)-\frac{\mu+\lambda}{2\mu+\lambda}\frac{c^2m^2}{a^2\gamma^2_{mn}}\Psi^{(1)}(\gamma_{mn}\zeta)\Big\}+D'_{mn}\Big\{\Big(1-\frac{\mu+\lambda}{2\mu+\lambda}\frac{c^2m_2}{a^2\gamma^2_{mn}}\Big)\\
&\times\phi^{(2)}(\gamma_{mn}\zeta)-\frac{\mu+\lambda}{2\mu+\lambda}\frac{c^2m^2}{a^2\gamma^2_{mn}}\Psi^{(2)}(\gamma_{mn}\zeta)\Big\}\Big]\cos\frac{\pi m}{a}x\cdot\cos\frac{n\pi}{b}y\\
&+\sum_n\sum_r\frac{a^2n}{2b\alpha_{nr}}\Big[A_{nr}\Big\{\frac{\mu}{2\mu+\lambda}\phi^{(1)}(\alpha_{nr}\xi)+\frac{\mu+\lambda}{2\mu+\lambda}\Psi^{(1)}(\alpha_{nr}\xi)\Big\}\\
&+A'_{nr}\Big\{\frac{\mu}{2\mu+\lambda}\phi^{(2)}(\alpha_{nr}\xi)+\frac{\mu+\lambda}{2\mu+\lambda}\Psi^{(2)}(\nu_{nr}\xi)\Big\}\Big]\cos\frac{m\pi}{a}x\cdot\cos\frac{n\pi}{b}y\\
&+\sum_m\sum_r\frac{b^2m}{2a\beta_{mr}}\Big[B_{mr}\Big\{\frac{\mu}{2\mu+\lambda}\phi^{(1)}(\beta_{mr}\eta)+\frac{\mu+\lambda}{2\mu+\lambda}\Psi^{(1)}(\beta_{mr}\eta)\Big\}\\
&+B'_{mr}\Big\{\frac{\mu}{2\mu+\lambda}\phi^{(2)}(\beta_{mr}\eta)+\frac{\mu+\lambda}{2\mu+\lambda}\Psi^{(2)}(\beta_{mr}\eta)\Big\}\Big]\cos\frac{m\pi}{a}x\cdot\cos\frac{r\pi}{c}z\\
&-\sum_m\sum_r\mu\frac{\beta_{mr}}{2}\Big\{H_{mr}\Big(1+\frac{b^2m^2}{a^2\beta^2_{mr}}\Big)\phi^{(1)}(\beta_{mr}\eta)+H'_{mr}\Big(1+\frac{b^2m^2}{a^2\beta^2_{mr}}\Big)\\
&\times\phi^{(2)}(\beta_{mr}\eta)\Big\}\cos\frac{m\pi}{a}x\times\cos\frac{r\pi}{c}z-\sum_n\sum_r\mu\frac{\alpha_{nr}}{2}\Big\{J_{nr}\Big(1+\frac{a^2n^2}{b^2\alpha^2_{nr}}\Big)\\
&\times\phi^{(1)}(\alpha_{nr}\xi)+J'_{nr}\Big(1+\frac{a^2n^2}{b^2\alpha^2_{nr}}\Big)\phi^{(2)}(\alpha_{nr}\xi)\Big\}\cos\frac{n\pi}{b}y\cdot\cos\frac{r\pi}{c}z\\
&+\sum_m\mu\frac{b}{4}\Big[B_{m0}\Big\{\frac{\mu}{2\mu+\lambda}\phi^{(1)}(\beta_{m0}\eta)+\frac{\mu+\lambda}{2\mu+\lambda}\Psi^{(1)}(\beta_{m0}\eta)\Big\}+B'_{m0}\\
&\times\Big\{\frac{\mu}{2\mu+\lambda}\phi^{(2)}(\beta_{m0}\eta)+\frac{\mu+\lambda}{2\mu+\lambda}\Psi^{(2)}(\beta_{m0}\eta)\Big\}\Big]\cos\frac{m\pi}{a}x+\sum_n\sum_r\frac{a}{4}\\
&\times\Big[A_{n0}\Big\{\frac{\mu}{2\mu+\lambda}\phi^{(1)}(\alpha_{n0}\xi)+\frac{\mu+\lambda}{2\mu+\lambda}\Psi^{(1)}(\alpha_{n0}\xi)\Big\}+A'_{n0}\Big\{\frac{\mu}{2\mu+\lambda}\phi^{(2)}(\alpha_{n0}\xi)\\
&+\frac{\mu+\lambda}{2\mu+\lambda}\Psi^{(1)}(\alpha_{n0}\xi)\Big\}\Big]\cos\frac{n\pi}{b}y-\sum_m\mu\frac{mb}{2a}\Big\{H_{m0}\phi^{(1)}(\beta_{m0}\eta)\\
&+H'_{m0}\phi^{(2)}(\beta_{m0}\eta)\Big\}\cos\frac{m\pi}{a}x-\sum_n\mu\frac{na}{2b}\Big\{J_{n0}\phi^{(1)}(\alpha_{n0}\xi)+J'_{n0}\phi^{(2)}(\alpha_{n0}\xi)\Big\}\\
&\times\cos\frac{n\pi}{b}y+\sum_n\mu\frac{c}{4}\Big\{D_{0n}\phi^{(1)}(\gamma_{0n}\zeta)+D'_{0n}\phi^{(2)}(\gamma_{0n}\zeta)\Big\}\cos\frac{n\pi}{b}y\\
&-\sum_r\mu\frac{br}{4c}\Big\{J_{0r}\phi^{(1)}(\beta_{0r}\eta)+J'_{0r}\phi^{(2)}(\beta_{0r}\eta)\Big\}\cos\frac{r\pi}{c}z+\sum_m\mu\frac{c}{4}
\end{aligned}\right\}\quad(73)
$$

$$\times \left\{F_{m0}\phi^{(1)}(\gamma_{m0}\zeta) + F'_{m0}\phi^{(2)}(\gamma_{m0}\zeta)\right\} \cos \frac{m\pi}{a} x - \sum_r \mu \frac{ra}{4c} \left\{H_{0r}\phi^{(1)}(\alpha_{0r}\xi)\right.$$
$$\left. + H'_{0r}\phi^{(2)}(\alpha_{0r}\xi)\right\} \cos \frac{r\pi}{c} x + \frac{1}{4}\left(H_{00} + J_{00}\right).$$

Eighteen unkown values A_{nr}, A'_{nr}, B_{mr}, B'_{mr}, D_{mn}, D'_{mn}, E_{nr}, E'_{nr}, F_{mn}, F'_{mn}, G_{mr}, G'_{mr}, H_{mr}, H'_{mr}, J_{nr}, J'_{nr}, K_{mn}, K'_{mn} can be all determined to satisfy the eighteen boundary conditions on the side planes. The terms involving the unkown values whose index n, m, or r are zero, indicate the behaviour in regard to the plane elasticity.

In virtue of Formulas (64)~(72), we could, for instance, investigate the differences between the simply supported plate and the hinged plate ; or to find out suitability of Kirchhoff's assumption for the edge reaction of the thin plate.

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